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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2675**

Unique Paper Code : **42351201**

Name of the Paper : **Calculus and Geometry**

Name of the Course : **B.Sc. (Prog.) Physical Sciences/ Mathematical Sciences (CBCS-LOCF)**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has *six* questions in all.

Attempt any *two* parts from each question.

All questions are compulsory.

Marks are indicated.

1. (*a*) Find the relative extrema using first derivative test of the function :

$$f(x) = 2x^3 - 9x^2 + 12x. \quad 6.5$$

(*b*) Find the open intervals on which the function $f(x) = x^4 - 8x^2 + 16$ is concave up or concave down. 6.5

(*c*) Sketch the graph of the function $y = x(x-1)^3$. 6.5

P.T.O.

2. (*a*) Evaluate the following limits using L'Hopital's rule :

(*i*) $\lim_{x \to 1} (2-x)^{\tan\left(\frac{\pi}{2}x\right)}$

(*ii*) $\lim_{x \to 0^+} \tan x \log x$. 6

(*b*) Sketch the curve by eliminating the parameter and indicate the direction of increasing t.

$$x = \cos 2t, \; y = \sin t, \; -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$ 6

(*c*) Sketch the graph of $r^2 = 4\cos 2\theta$ in polar coordinates. 6

3. (*a*) Find the volume of the ellipsoid formed by the revolution of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

about the major axis. 6

(*b*) Find the area of the parabola $y^2 = 4ax$ bounded by its latus rectum. 6

(*c*) Find the length of the curve :

$$y = \log \frac{e^x - 1}{e^x + 1},$$

as x varies from 1 to 2. 6

4. (*a*) Obtain the reduction formula for $\int \sin^n x \, dx$ (n being a positive integer) and hence evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^5 x \, dx.$$ 6.5

(*b*) Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx.$$ 6.5

(*c*) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then show that for

$$n > 1, \ (I_n + I_{n-2}) = \frac{1}{n-1}.$$

Hence deduce the value of I_5. 6.5

5. (*a*) Rotate the coordinate axes to remove *xy* term from the equation :

$$x^2 + 2\sqrt{3}\, xy + 3y^2 + 2\sqrt{3}x - 2y = 0,$$

and trace the conic. 6.5

(*b*) Sketch the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ and label the foci and draw the directrices. 6.5

(*c*) Find an equation of the parabola that is symmetric about the *y*-axis, has its vertex at the origin and passes through the point (5, 2). Also write its inflection property. 6.5

6. (*a*) Show that the function $\overrightarrow{r(t)} = (2\sin t, 2\cos t, \sqrt{5})$ has constant length and is orthogonal to its derivative. 6

(*b*) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, then prove that :

$$\bar{\nabla} \cdot \left(\frac{\bar{r}}{r}\right) = \frac{2}{r}.$$ 6

(*c*) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is time. Find its velocity and acceleration at t = 1 in the direction of vector $\bar{r} = 2\hat{i} + 2\hat{j} - \hat{k}$. 6

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5803**

Unique Paper Code : **2352011202**

Name of the Paper : **Calculus-DSC 5**

Name of the Course : **Bachelor of Science (Honours Course) Mathematics NEP UGCF-2022**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *three* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Using $\varepsilon - \delta$ definition of limit, show that :

$$\lim_{x \to c} \frac{1}{x} = \frac{1}{c},$$

if $c > 0$.

(*b*) Show that if $f : (a, \infty) \to \mathbf{R}$ $(a > 0)$ is such that

$$\lim_{x \to \infty} x f(x) = \mathrm{L},$$

where $\mathrm{L} \in \mathbf{R}$, then

$$\lim_{x \to \infty} f(x) = 0.$$

P.T.O.

(*c*) Let $A \subset \mathbf{R}$ and $f : A \to \mathbf{R}$ has a limit at $c \in \mathbf{R}$, where c is a cluster point of A. Then prove that f is bounded on some neighbourhood of c.

(*d*) Give example of functions f and g such that f and g do not have limits at a point c, but both $f + g$ and fg have limits at c.

2. (*a*) If f is continuous on a closed interval $[a, b]$, then prove that f is uniformly continuous on $[a, b]$.

(*b*) Show that the function $f(x) = x^2$ is uniformly continuous on $[-4, 4]$.

(*c*) Show that :

$$\lim_{x \to \infty} \frac{x+2}{\sqrt{x}} \ (x > 0)$$

does not exist in **R**.

(*d*) Use $\varepsilon - \delta$ definition of limit to show that :

$$\lim_{x \to 6} \frac{x^2 - 3x}{x+3} = 2.$$

3. (*a*) Let

$$f(x) = \begin{cases} x^2, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Calculate f' on **R**. Is f' continuous as well as differentiable on **R** ? Justify your answer.

(*b*) Let $f(x) = \sin |x|$, where $x \in \mathbf{R}$. Determine the set of points where f is not differentiable. Justify your answer.

(c) If $f : \mathbf{R} \to \mathbf{R}$ is differentiable at $a \in \mathbf{R}$, then show that :

(i) $\lim_{h \to 0} \dfrac{f(a+h)-f(a)}{h} = f'(a)$,

(ii) $\lim_{h \to 0} \dfrac{f(a+h)-f(a-h)}{2h} = f'(a)$.

(d) Suppose f is defined on an open interval (a, b) Let $x_0 \in (a, b)$. If f assumes its maximum at x_0 and if f is differentiable at x_0, then show that $f'(x_0) = 0$.

4. (a) State and prove Rolle's theorem.

(b) Let f be defined on $\mathbf{R}$ and suppose that $|f(x) - f(y)| \leq |x - y|^3$ for all $x, y \in \mathbf{R}$. Prove that f is a constant function on $\mathbf{R}$.

(c) Show that $\sin x \leq x$ for all $x \geq 0$.

(d) State Intermediate value theorem for derivatives. Suppose f is differentiable on $\mathbf{R}$ and $f(0) = 1$, $f(1) = 2$ and $f(2) = 2$.

(i) Show that $f'(x) = \dfrac{1}{2}$ for some $x \in (0, 2)$,

(ii) Show that $f'(x) = \dfrac{1}{3}$ for some $x \in (0, 2)$.

5. (a) If

$$y = \log\left(x + \sqrt{1 + x^2}\right),$$

show that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

(b) Trace the curve :

$$4x^2 = y^2(x^2 - 4).$$

(c) Determine the intervals of concavity and points of inflexion (if any) of the curve :

$$y = 2x^4 - 3x^2 + 2x + 1.$$

(d) State Taylor's Theorem. Find the Taylor's series expansion of $\sin 2x$ about $x = 0$.

6. (a) Trace the curve :

$$r = 2 \sin 2\theta.$$

(b) Determine the position and nature of the double points of the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0.$$

(c) Find the horizontal asymptotes of the graph of the function given by

$$y = x^5\left[\sin\frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3}\right].$$

(d) Find the equations of tangents at origin to the curve :

$$x^4 + y^4 = a^2(x^2 - y^2).$$

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5967**

Unique Paper Code : **2352571201**

Name of the Paper : **Elementary Linear Algebra**

Name of the Course : **B.A./B.Sc. (Prog.) with Mathematics as Non-Major/Minor-DSC**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of simple calculators is allowed.

1. (*a*) Define $\|x\|$ for any vector x in $\mathbf{R}^n$. Prove that :

$$x \cdot y = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2)$$

for any vectors x and y in $\mathbf{R}^n$. 2+5.5

(*b*) Use the Gaussian elimination method to solve the system of linear equations :

$$x_1 + x_2 + x_3 = 5$$
$$2x_1 + x_2 - x_3 = 2$$
$$2x_1 - x_2 + x_3 = 2.$$

7.5

P.T.O.

(c) Find the rank of the matrix :

$$A = \begin{bmatrix} 1 & 3 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$ 7.5

2. (a) Determine whether the vector [7, 1, 18] is in the row space of the matrix :

$$\begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$ 7.5

(b) Use the Gauss-Jordan method to convert the matrix A to reduced row echelon form :

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 5 \end{bmatrix}.$$ 7.5

(c) Find the eigenvalues and the corresponding eigenspaces for the following matrix :

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & -4 \end{bmatrix}.$$ 7.5

3. (a) Use the Diagonalization Method to determine whether the following matrix A is diagonalizable. If so, specify the matrices D and P. Check your work by verifying that $D = P^{-1}AP$.

$$A = \begin{bmatrix} -3 & 3 & -1 \\ 2 & 2 & 4 \\ 6 & -3 & 4 \end{bmatrix}.$$ 7.5

(b) State True *or* False for the following statements. Give a justification for your choice :

(i) The set of polynomials of degree seven is a vector space under the operations of usual addition and scalar multiplication of polynomials.

(ii) The set of non-singular 2×2 matrices, under the operations of usual matrix addition and scalar multiplication, is a vector space. 3.5+4

(c) Show that the set of vectors of the form $[a, b, 0, c, a - 2b + c]$ in $\mathbf{R}^5$ forms a subspace of $\mathbf{R}^5$ under the operations of vector addition and scalar multiplication. 7.5

4. (a) Use the Simplified Span Method to find a simplified general form for all vectors in span (S), where S is a subset of $\mathbf{M}_{2 \times 2}$ given by

$$\left\{\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\right\}.$$ 7.5

(b) Determine whether the given subset of P is linearly independent :

$$\{4x^2 + 2, x^2 + x - 1, x, x^2 - 5x - 3\}.$$ 7.5

(c) Define a basis for a vector space. Prove that the following set is a basis of $\mathbf{M}_{2 \times 2}$:

$$\left\{\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 5 & -2 \\ 0 & -3 \end{bmatrix}\right\}.$$ 2+5.5

5. (a) Define $T : M_{2 \times 2} \to M_{2 \times 2}$ given by $T(A) = A^T$, where $M_{2 \times 2}$ is the vector space of all 2×2 matrices over $\mathbf{R}$. Is T a linear operator? Find range T and kernel T. Is T an isomorphism ? 2+4+1.5

(b) Find the matrix of the linear transformation $T : P_3 \to P_2$ given by $T(p(x)) = p'(x)$ with respect to the standard ordered bases of P_3 and P_2 respectively. 7.5

(c) Find the range and kernel of the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined as $T(a, b, c) = (a - b, 2a)$. Verify the Dimension Theorem for T. 2.5+2.5+2.5

6. (a) Are $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined as $T(x, y) = (0, y)$ and $S : \mathbf{R}^2 \to \mathbf{R}^2$ defined as $S(x, y) = (\sin x, y)$ linear ? Justify your answer. 3.5+4

(b) Let A be a fixed non-singular $n \times n$ matrix. Show that $T : M_{n \times n} \times M_{n \times n}$ given by $T(B) = AB$ is an isomorphism. 7.5

(c) Define $T : M_{2 \times 2} \to \mathbf{R}$ as $T(A) = \text{Trace } A$. Is T a linear transformation? Find the matrix of T with respect to the standard ordered bases of $M_{2 \times 2}$ (over $\mathbf{R}$) and $\mathbf{R}$ (over $\mathbf{R}$) respectively. 2.5+5

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5377**

Unique Paper Code : **2354001202**

Name of the Paper : **Introduction to Linear Algebra**

Name of the Course : **Common Prog Group**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

1. (*a*) Let x and y be vectors in $\mathbf{R}^n$. Prove that $\|x + y\| \leq \|x\| + \|y\|$.

(*b*) Solve the following system of linear equations using Gauss-Elimination method :

$$x_1 + 2x_2 + 3x_3 = 1$$

$$x_1 + 3x_2 + 5x_3 = 2$$

$$2x_1 + 5x_2 + 9x_3 = 3.$$

(*c*) Solve the following system of linear equations using Gauss-Jordon method :

$$2x_1 + 3x_2 - x_3 = 9$$

$$x_1 + x_2 + x_3 = 9$$

$$3x_1 - x_2 - x_3 = -1.$$

P.T.O.

2. (a) Define rank of a matrix. Find the rank of the following matrix :

$$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}.$$

(b) Find the characteristic polynomial of the matrix :

$$A = \begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}.$$

Also, find all the eigenvalues and eigenspace for any *one* eigenvalue.

(c) Is the matrix :

$$A = \begin{bmatrix} -3 & -1 & -2 \\ -2 & 16 & -18 \\ 2 & 9 & -7 \end{bmatrix}$$

diagonalizable ? Justify.

3. (a) Show that the set $\mathbf{R}^2$, with the usual scalar multiplication but with vector addition given by $[x, y] \oplus [w, z] = [x + y, 0]$ is not a vector space.

(b) Use Simplified Span Method to find a simplified general form for all the vectors in Span(S), where :

$S = [x^3 - 1, x^2 - x, x - 1]$ is the subset of $\mathbf{P}_3$.

(c) Show that the subset $\{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$ of $\mathbf{R}^3$ forms a basis of $\mathbf{R}^3$.

4. (a) Prove or disprove that the subset :

$$S = \left\{ \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}; a, b \in \mathbf{R} \right\}$$

is a subspace of M_{22} under usual matrix operations.

(b) Use Independent Test Method to find whether the set :

$$S = \{[2, -1, 3], [4, -1, 6], [-2, 0, 2]\}$$

of vectors is linearly independent or linearly dependent.

(c) Determine whether the vector X = [7, 1, 18] is in the row space of the matrix :

$$A = \begin{bmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{bmatrix}.$$

If so, then express [7, 1, 18] as a linear combination of the rows of A.

5. (a) Define the linear transformation. Determine whether the following function is a linear transformation :

$$T_1 : M_{32} \to P_4 \text{ given by } T_1 \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix} = a_{11}x^4 - a_{21}x^2 + a_{31}.$$

(b) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear operator that performs a counterclockwise through an angle 30°. Find the matrix representation for T with respect to the ordered basis B = {[1, 1], [2, 3]} for R^2.

P.T.O.

(c) Let $T : P_2(\mathbf{R}) \to P_3(\mathbf{R})$ be given by :

$$T(p(x)) = \int p(x)\,dx.$$

Find the matrix for T with respect to standard bases for $P_2(\mathbf{R})$ and $P_3(\mathbf{R})$. Use this matrix to calculate $T(3x^2 - 2x + 5)$ by matrix multiplication.

6. (a) Suppose $L : \mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator with :

$$L([1, 2]) = [1, 1] \text{ and } L([2, 0]) = [2, -4].$$

Give a formula for $L([x, y])$ for any $[x, y] \in \mathbf{R}^2$. Also, find a basis for Ker(L) and Range(L).

(b) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -1 & 5 \\ -2 & 3 & -13 \\ 3 & -3 & 15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for Ker(T) and a basis for Range(T). Also, verify that :

$$\dim(\mathrm{Ker}(T)) + \dim(\mathrm{Range}(T)) = \dim(\mathbf{R}^3).$$

(c) Let $T : M_{23}(\mathbf{R}) + M_{22}(\mathbf{R})$ be the linear transformation given by :

$$T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a+b & a+c \\ d+e & d+f \end{bmatrix}.$$

Show that T is onto but not one-to-one.

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5783**

Unique Paper Code : **2352011201**

Name of the Paper : **Linear Algebra**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **II / DSC**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Prove that if x, y and z are mutually orthogonal vectors in $\mathbf{R}^n$, then

$$\|x + y + z\|^2 = \|x\|^2 + \|y\|^2 + \|z\|^2.$$

Also, verify the same for the vectors in $\mathbf{R}^4$, where $x = (1, 0, -1, 0)$, $y = \left(1, \sqrt{2}, 1, 1\right)$ and $z = \left(1, -\sqrt{2}, 1, 0\right)$.

(*b*) Solve the following system of linear equations using Gaussian Elimination Method :

$$3x_1 - 2x_2 + 4x_3 = -54$$
$$-x_1 + x_2 - 2x_3 = 20$$
$$5x_1 - 4x_2 + 8x_3 = -83.$$

(*c*) Using the Gauss-Jordon method, find the minimal integer values for the variables that will balance the following chemical equation :

$a(HNO_3) + b(HCl) + c(Au) \rightarrow d(NOCl) + e(HAuCl_4) + f(H_2O)$.

P.T.O.

2. (a) Define a linear combination of the vectors in $\mathbf{R}^n$. Express the vectors [1, 11, –4, 11] as a linear combination of the vectors x = [2, –4, 1, –3], y = [7, –1, –1, 2] and z = [3, 7, –3, 8].

(b) Consider the matrix :

$$A = \begin{bmatrix} 5 & -8 & -12 \\ -2 & 3 & 4 \\ 4 & -6 & -9 \end{bmatrix}$$

(i) State the Cayley-Hamilton Theorem.

(ii) Find the eigenvalues and eigenvectors of A.

(iii) Check whether the matrix A is diagonalizable or not ?

(c) Let

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 1 \\ -2 & 1 & 5 \\ 3 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & -5 \\ 2 & 3 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

Compute R(AB) and (R(A))B to verify that they are equal, i.e., R(AB) = (R(A))B, for the operations :

(i) R : <3> ← –3 <2> + <3>,

(ii) R : <2> ↔ <3> .

3. (a) Let $\mathbf{R}^2 = \{(a_1, a_2) : a_1, a_2 \in \mathrm{R}\}$. Prove that $\mathbf{R}^2$ is a vector space over $\mathbf{R}$ with addition and scalar multiplication defined as;

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$

$$a(a_1, a_2) = (\alpha a_1, \alpha a_2) \text{ where } (a_1, a_2), (b_1, b_2) \in \mathbf{R}^2, \alpha \in \mathbf{R}.$$

(b) Define basis of a vector space. Prove that the following subset S of $\mathbf{R}^3$ is a spanning set of $\mathbf{R}^3$, where $S = \{(2, 2, 3), (-1, -2, 1), (0, 1, 0)\}$.

(c) Do the polynomials $x^3 - 2x^2 + 1$, $4x^3 - x + 3$ and $3x - 2$ generates $P_3(\mathbf{R})$?

4. (a) If $\{v_1, v_2, \ldots\ldots, v_n\}$ is a basis of a vector space V(F), then prove that every element of V can be uniquely expressed as a linear combination of $v_1, v_2, \ldots\ldots, v_n$.

(b) Define linearly dependent and independent set in $\mathbf{R}^3$. Show that set of vectors $\{(2, -2, 3), (0, -4, 1), (3, 1, -4)\}$ in $\mathbf{R}^3$ is linearly dependent.

(c) Let W be a subspace of a finite-dimensional vector space V. Prove that W is finite-dimensional and $\dim(W) \leq \dim(V)$. Moreover, if $\dim(W) = \dim(V)$, then $V = W$

5. (a) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ is a function. For each of the following parts, state why T is not a linear transformation.

(i) $T(a, b) = (1, b)$

(ii) $T(a, b) = (a, b^2)$

(iii) $T(a, b) = (\sin a, 0)$

(iv) $T(a, b) = (|a|, b)$

(v) $T(a, b) = (1 + a, b)$.

(b) Let V, W and Z be finite-dimensional vector spaces with ordered basis α, β and γ, respectively. Let $T : V \to W$ and $U : W \to Z$ be linear transformations. Then $[UT]_\alpha^\beta = [U]_\alpha^\beta [T]_\alpha^\beta$.

(c) Prove that two same dimensional finite vector spaces over the same field are isomorphic.

6. (a) Let V and W be vector spaces over same field F, and suppose that $\{v_1, v_2, ..., v_n\}$ is a basis for V. For $w_1, w_2, ..., w_n$ in W, there exists exactly one linear transformation $T : V \to W$ such that $T(v_i) = w_i$ for $i = 1, 2, ..., n$.

(b) Let $L : \mathbf{M}_{2 \times 2}(\mathbf{R}) \to \mathbf{R}^3\ (\mathbf{R})$ be a linear transformation given by

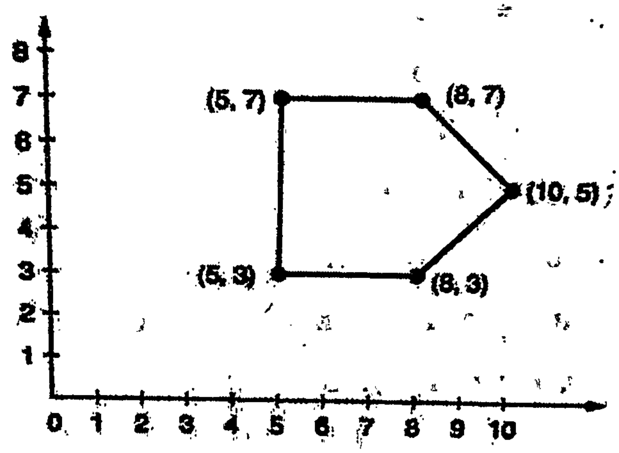
$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a - c + 2d,\ 2a + b - d,\ -2c + d] \text{ with}$$

$$B = \left(\begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}\right);$$

$C = ([7, 0, -3], [2, -1, -2], [2, 0, 1])$.

Check whether B and C are bases for $\mathbf{M}_{2 \times 2}(\mathbf{R})$ and $\mathbf{R}^3\ (\mathbf{R})$, respectively, if yes, then find matrix A for L with respect to the given bases B and C.

(c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



(i) Translation along the vector [4, –2]

(ii) Rotation about the origin through $\theta = 30°$

(iii) Reflection about the line $y = 3x$.

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5823**

Unique Paper Code : **2352011203**

Name of the Paper : **Ordinary Differential Equations**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II/DSC**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any *two* parts from each question.

Each part carries **7.5** marks.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Solve the initial value problem :

$(2y \sin x \cos x + y^2 \sin x)\,dx + (\sin^2 x - 2y \cos x)\,dy = 0, y(0) = 3.$

(*b*) Solve :

$$(x^2 - 3y^2)\,dx + 2xy\,dy = 0.$$

(*c*) Solve :

$$x^2y'' = 2xy' + (y')^2$$

by reduction of order.

2. (*a*) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants ?

P.T.O.

(b) A metal bar at a temperature of 100°F is placed in a room at a constant temperature of 0°F. If after 20 minutes the temperature of the bar is half, find an expression for the temperature of the bar at any time.

(c) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei in the sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 200 years. What percentage of the original radioactive nuclei will remain after 1000 years ?

3. (a) Show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$$

Find the particular solution satisfying the initial condition,

$$y(0) = 0, \ y'(0) = 0.$$

(b) Solve the differential equation using the method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$

(c) Find the general solution of the differential equation using the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x.$$

4. (a) Use operator method to find the general solution of the following linear system :

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dy}{dt} + 2x + y = 3e^t.$$

(b) Solve the initial value problem, assuming $x > 0$

$$x^3 \frac{d^3y}{dx^3} - x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = x^3.$$

(c) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ m and initial velocity $v_0 = -10$ m/s. Find the amplitude, period and frequency of the resulting motion.

5\. (a) Define carrying capacity. Derive the logistic equation :

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right)$$

where r and K are reproduction rate and carrying capacity respectively.

(b) Suppose a soft drink manufacturing company has a 1000 litre tank containing sugar water solution. Initially s_0 kg of sugar is dissolved. Sugar solution flows into the tank at the rate of 10 litre/min with concentration $cin(t)$ kg/l. Assume that the solution in the tank is thoroughly mixed and the sugar solution flows out at the same rate at which it flows in i.e. the volume of sugar water mixture in the tank remains constant. Find a differential equation for the amount of sugar in the tank at any time t. Also solve it.

(c) Suppose a population can be modelled using the differential equation :

$$\frac{dx}{dt} = 0.2x - 0.001x^2$$

with the initial population size of $x_0 = 100$ at a time step of 1. Find the predicted population after two months.

6. (*a*) The given differential equation :

$$\frac{dN}{dt} = aN\left(1 - \frac{N}{N_T}\right)$$

represents the model for spread of technology where $N(t)$ is the number of ranchers who adopted an improved pasture technology in Uruguay and N_T is the total population of ranchers. It is assumed that rate of adoption is proportional to both the number who have adopted the technology and the fraction of population of ranchers who have not adopted the technology.

(*i*) Which term corresponds to the fraction of the population who have not yet adopted the improved pasture technology ?

(*ii*) According to the Banks (1994), $N_T = 17015$, $a = 0.490$ and $N_0 = 141$. Determine how long it takes for the improved pasture technology to spread to 80% of the population.

(*b*) Derive the epidemic model of influenza

$$\frac{dS}{dt} = -\beta\, SI,\ \frac{dI}{dt} = \beta\, SI - \gamma I,\ \frac{dS}{dt} = \gamma I$$

where β and γ are positive constants.

(*c*) Define equilibrium points. Find all equilibrium points of Predator-Prey model

$$\frac{dX}{dt} = \beta_1 X\left(1 - \frac{X}{K}\right) - c_1 XY,\ \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

where the positive constants c_1, c_2 are interaction parameter, α_2 is predator per capita death rate and β_1 is prey per capita birth rate. Discuss the direction of trajectories of Predator-Prey model.

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5874**

Unique Paper Code : **2352572401**

Name of the Paper : **Abstract Algebra**

Name of the Course : **B.A./B.Sc. (Programme)**

Type of the Paper : **DSC**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting *two* parts from each question.

Both parts of a question to be attempted together.

All questions carry equal marks.

Use of calculator is not allowed.

1. (*a*) Define a group. Prove that in a group G, left and right cancellation laws hold true. 2+5.5

(*b*) Show that the set

$$G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a \in \mathbf{R}, a \neq 0 \right\}$$

is a group under matrix multiplication. 7.5

(c) Describe each symmetry in D_3 (the set of symmetries of an equilateral triangle). Also construct its coresponding Cayley table. 3.5+4

2. (a) Let G be an Abelian group and H, K be subgroups of G. Then prove that :

$$HK = \{hk \mid h \in H, k \in K\}$$

is a subgroup of G. 7.5

(b) Define order of an element in a group G. Find the order of each element in the group U(15). 2+5.5

(c) State and prove One-Step Subgroup Test. 2+5.5

3. (a) Define a cyclic group. Let $|a| = 30$, then find $\langle a^{26} \rangle$, $\langle a^{17} \rangle$, $\langle a^{18} \rangle$ and $|a^{26}|$, $|a^{17}|$ and $|a^{18}|$. 1.5+3+3

(b) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix}$, $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$

Compute each of the following :

(i) $\alpha\beta$

(ii) $\beta\alpha$

(iii) $o(\alpha)$ and $o(\beta)$. 2+2+3.5

(c) Let $G = S_3$ and $H = \{(1), (13)\}$. Then find left and right cosets of H in G. 7.5

4. (*a*) Let G be group and H be a subgroup of G. Then for a, $b \in$ G, either $a\text{H} = b\text{H}$ or $a\text{H} \cap b\text{H} = \phi$. 7.5

(*b*) Prove that the center Z(G) of a group G is a normal subgroup of G. Let H = {(1), (12)}. Is H normal in S_3 ? 4+3.5

(*c*) Define kernel of a group homomorphism $\phi : G \to \bar{G}$. Show that *Ker* ϕ is a subgroup of group G. 1.5+6

5. (*a*) Find the unity in the ring {0, 2, 4, 6, 8} under addition and multiplication modulo 10. In Z_6, show that $4 | 2$ (4 divides 2) and in Z_8, show that $3 | 7$ (3 divides 7). 2.5+5

(*b*) Define characteristic of a ring. Prove that characteristic of an integral domain is either '0' or prime. 2.5+5

(*c*) Show that every non-zero element of Z_n is a unit or a zero divisor. 7.5

6. (*a*) Describe the factor ring Z/6Z. Is Z/6Z a field ? Justify. 3+4.5

(*b*) Determine all ring homomorphisms from Z_n to itself. 7.5

(*c*) Prove that the only ideals of a field F are {0} and F itself. 7.5

(b) Test for the convergence of the series whose nth term is

$$\frac{\sqrt{n+1}-\sqrt{n-1}}{n}.$$

(c) Show that the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\log(n+1)}$$

is conditionally convergent.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4765 **J**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **Common Program Group: GE**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

2. Attempt all questions by selecting **two** parts from each question.

3. Part of the questions to be attempted together.

4. **All** questions carry equal marks.

1. (a) Define infimum and supremum of a set. Find the infimum and supremum, if exist, of the following sets:

(i) $\left\{2+\frac{1}{n}: n \in \mathbb{N}\right\}$.

(ii) $\left\{\frac{x+1}{x}: x > 2\right\}$.

(b) Let F be an Archimedean ordered field, $A \subseteq F$, and $u \in F$. Prove that $u = \sup A$ if and only if for all $\varepsilon > 0$,

(i) for all $x \in A$, $x < u + \varepsilon$, and

(ii) there exists $x \in A$, such that $x > u - \varepsilon$.

(c) Define a field. Prove that the set $\mathbb{Z}$ of integers with ordinary addition and multiplication is not a field.

is not convergent by using the Cauchy criterion for convergence of series.

(c) Determine whether the Geometric series

$$\sum_{n=0}^{\infty} \frac{3^n + 4^n}{5^n}$$

converges or diverges. Find the sum of the series if it converges.

6. (a) State the Integral Test for convergence of a series. Use it to show that

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

diverges.

(c) Find the limit of the sequence $\{x_n\}$, where x_n is given by

(i) $x_n = \frac{n^2}{(n)!}$

(ii) $x_n = \frac{1+2+3+\cdots+n}{n^2}$

5. (a) Prove that if the series $\sum a_n$ converges, then $\lim\limits_{n\to\infty} a_n = 0$. Give an example of a divergent series such that $\lim\limits_{n\to\infty} a_n = 0$. What do you conclude?

(b) Show that the Harmonic series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

2. (a) In any ordered field F, prove the following properties:

(i) $-|x| \le x \le |x|$.

(ii) $\max\{x, y\} = \frac{x+y+|x-y|}{2}$ and $\min\{x, y\} \frac{x+y-|x-y|}{2}$.

(b) State completeness property of an ordered field. Prove that in a complete ordered field F, $\sup(A \cup B) = \max\{\sup A, \sup B\}$, where A and B are bounded above subsets of F.

(c) (i) Let

$$x_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}}$$

for $n \in \mathbb{N}$. Show that the sequence $\{x_n\}$

converges. Find its limit.

(ii) Give an example of sequences $\{x_n\}$ and $\{y_n\}$ for which $\lim_{n\to\infty} x_n = 0,$, but $\lim_{n\to\infty} x_n y_n \neq 0$.

3. (a) Define the convergence of a sequence. Using the definition of convergence, evaluate

$$\lim_{n\to\infty} \frac{3n^2 + 4n}{n^2 + 5}.$$

Find the value of the natural number n_0 for $\in = 0.01$

(b) Let $X = \{x_n\}$ and $Y = \{y_n\}$ be sequences of real numbers that converge to x and y, respectively. Prove that the sequence $X + Y$ converges to $x + y$.

(c) Let $\{x_n\}$ be a sequence of real numbers defined

inductively by $x_1 = \sqrt{7}$ and

$$x_{n+1} = \sqrt{7 + x_n},$$

for all $n \in \mathbb{N}.$. Prove that $\{x_n\}$ converges, and find its limit.

4. (a) Prove that

$$\lim_{n\to\infty} n^{1/n} = 1.$$

(b) Define Cauchy sequence of real numbers. Further, prove that the sequence

$$\{x_n\} = \left\{\frac{n}{n^2 + 1}\right\}$$

is a Cauchy sequence.

Where $F = (x^5 + 10x\,y^2z^2)\,\hat{i} + (y^5 + 10y\,x^2z^2)\,\hat{j} + (z^5 + 10z\,x^2y^2)\,\hat{k}$, and S is the closed hemispherical surface $z=\sqrt{1-x^2-y^2}$ together with the disc $x^2 + y^2 \leq 1$ in $xy-$plane and N is the outward unit normal vector field.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5594 **J**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : **B.A. / B.Sc. (H)**

Semester : IV (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator is **NOT** allowed.

1. (a) Let $f(x,y) = \dfrac{\sin\left(x^2+y^2\right)}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Find the value of $f(0,0)$ for which $f(x,y)$ is continuous at $(0,0)$.

(b) Compute the slope of the tangent line to the graph of $f(x, y) = \dfrac{x^2+y^2}{xy}$ at $P(1, -1, -2)$ in the direction parallel to

(i) XZ plane

(ii) YZ plane.

(c) Find $\dfrac{\partial w}{\partial r}$ where $w = e^{2x-y+3z^2}$ and $x = r + s - t$, $y = 2r - 3s$, $z = \cos(rst)$.

6. (a) Evaluate the surface integral

$$\iint_S g\, dS$$

where $g(x,y,z) = xz + 2x^2 - 3xy$ and S is that portion of the plane $2x - 3y + z = 6$ that lies over the unit square R: $2 \le x \le 3$, $2 \le y \le 3$.

(b) Evaluate

$$\oint_C \left(\frac{1}{2}y^2\, dx + z\, dy + x\, dz\right)$$

where C is the curve of intersection of the plane $x + z = 1$ and the ellipsoid $x^2 + 2y^2 + z^2 = 1$, oriented counterclockwise.

(c) Use the divergence theorem to evaluate

$$\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$$

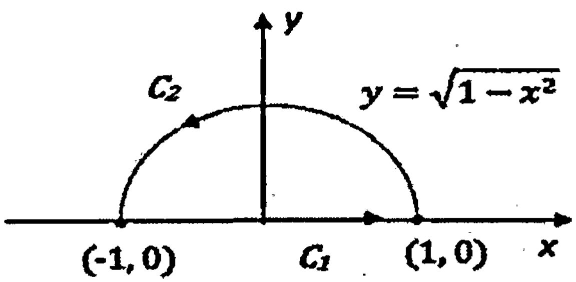
$$F = \left(e^x \sin y - y\right)\hat{i} + \left(e^x \cos y - x - 2\right)\hat{j}$$

is conservative and hence find a scalar potential function f for F.

(c) Verify Green's theorem for the line integral

$$\oint_C (-y\,dx + x\,dy)$$

where C is the closed path shown in the figure below



Path C

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point $P(2, 2, -2)$, find the unit vector pointing in the direction of most rapid increase of $f(x, y, z)$.

(b) Find all the critical points of $f(x,y) = (x-1)(y-1)(x+y-1)$ and classify each as a point of relative maximum, point of relative minimum or a saddle point.

(c) Find the minimum value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to $4x^2 + 2y^2 + z^2 = 4$.

3. (a) (i) Find the volume of the solid bounded below by the rectangle R in the xy-plane and above by the graph of $z = 2x + 3y$; R: $0 \le x \le 1$, $0 \le y \le 2$.

(ii) Evaluate $\int_0^1 \int_x^1 e^{y^2}\,dy\,dx$.

(b) (i) Sketch the region of integration and write an equivalent integral with the order of

integration reversed $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dxdy$.

(ii) Use a double integral for finding the volume of the solid region bounded above by the paraboloid $z = 6 - 2x^2 - 3y^2$ and below by the plane $z = 0$.

(c) Use a double integral to find the area bounded by the curve $r = 1 + \sin\theta$.

4. (a) Use cylindrical co-ordinates to compute the integral

$\iiint_D z\left(x^2+y^2\right)^{-1/2} dxdydz$ where D is the solid bounded above by the plane $z = 2$ and bounded below by the surface $2z = x^2 + y^2$.

(b) (i) Compute the iterated triple integral

$$\int_0^1 \int_0^y \int_0^{\ln y} e^{z+2x}\, dzdxdy.$$

(ii) Evaluate the iterated integral

$$\int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\cos\varphi} \rho^2 \sin\varphi\, d\rho d\theta d\varphi.$$

(c) Evaluate $\iint_R e^{(2y-x)/(y+2x)}\, dA$

where R is the trapezoid with vertices (0,2), (1,0), (4,0) and (0,8).

5. (a) A wire has the shape of the curve

$x = \sqrt{2}\sin t \quad y = \cos t \quad z = \cos t \quad \text{for } 0 \le t \le \pi$

If the wire has density $\delta(x, y, z) = xyz$ at each point (x, y, z), what is its mass?

(b) Show that the vector field

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5673**

Unique Paper Code : **2352012403**

Name of the Paper : **Numerical Analysis**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory. Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Define order of convergence of a sequence $\{p_n\}$ which converges to a number p. The sequence generated by the formula $x_{n+1} = \dfrac{x_n^3 + 3x_n a}{3x_n^2 + a}$ converges to $\sqrt{a}$. Determine the order of convergence and the asymptotic error constant.

(*b*) Perform four iterations of Bisection method to approximate the root of the equation $x^4 - x - 10 = 0$, in the interval (1, 2).

(*c*) Explain geometrically the Newton's method to determine a root p of an equation $f(x) = 0$. Hence, using initial approximation $p_0 = 2$, perform two iterations to obtain approximation to the value of $\sqrt[3]{17}$.

P.T.O.

2. (a) Use Newton's method to approximate the root of the equation $\ln(1 + x) - \cos(x) = 0$, in the interval (0, 1). Use error tolerance = 10^{-4}, and initial approximation $p_0 = 0.5$.

(b) Define a fixed point of a function. Let g be a continuous function on the closed interval $[a, b]$, with $g : [a, b] \rightarrow [a, b]$. Furthermore, suppose that g' is continuous on the open interval (a, b), and there exists a positive constant k such that $|g'(x)| \leq k < 1$, $\forall x \in (a, b)$. Prove that if $g'(p) \neq 0$, then for any $p_0 \in [a, b]$, the sequence $\{p_n\}$, where $p_n = g(p_{n-1})$, converges only linearly to the fixed point p.

(c) Perform four iterations of Secant method to approximate the root of $x^4 - 7x^2 + 3 = 0$, in the interval (0, 1), using initial approximations $p_0 = 0$ and $p_1 = 1$.

3. (a) For the following system of equations :

$$8x + 5y + 2z = 8,$$

$$6x + 9y + 2z = 11,$$

$$4x + 3y + 8z = 1,$$

starting with the initial approximation as (0, 0, 0), using Gauss Jacobi method, find approximate solution by performing three iterations.

(b) For the following system of equations :

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18,$$

$$2x - 3y - 20z = 25,$$

starting with initial approximation as (0, 0, 0), using Gauss Seidel method, find the approximate solution by performing three iterations.

(c) Find the LU decomposition by taking $u_{ii} = 1$ for the matrix :

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}.$$

4. (a) Construct the Newton form of interpolating polynomial for a function f passing through the points (0, 1), (1, 3) and (3, 55). Hence, evaluate $f(2.5)$.

(b) Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use Lagrange interpolation to calculate an approximate value for $f(1.04)$ and obtain a bound on the truncation error at $x = 1.04$.

(c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the data :

x	$f(x)$
1	3
2	7
4	21
8	73

5. (a) Derive the central difference approximation to the second order derivative of a function f at a point $x = x_0$

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}.$$

(*b*) Verify that the following difference approximation

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$

for the first order derivative provides the exact value of the derivative, regardless of the values of h, for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

(*c*) By using Trapezoidal rule, approximate the value of the integral $\int_2^3 (x + \ln x)\,dx$. Further, calculate the absolute error, theoretical error bound and compare them.

6. (*a*) Define the degree of precision of a quadrature rule. Further, calculate the degree of precision of Simpson's rule.

(*b*) Use Euler's method to determine the approximate solution of the initial value problem $y'(x) = xy^3 - y$, $0 \le x \le 2$ such that $y(0) = 2$, by taking the step size as $h = 1/2$.

(*c*) Use Modified Euler's method to determine the approximate solution of the initial value problem : $y'(x) = 3x - y/x$, $1 \le x \le 2$, such that $y(1) = 2$, by taking the step size as $h = 1/2$.

(c) Define a power series. Determine the radius of convergence and exact interval of convergence of the power series $\sum \frac{x^n}{n^n}$. (6,6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2677 **J**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **CBCS-LOCF: B.Sc. (Prog) Physical Sciences / Mathematical Sciences**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Define Supremum of a non-empty bounded subset of R.

Let S be a non-empty bounded above subset of R and let 'a' $\in$ R, such that $a + S = \{a + s: s \in S\}$. Prove that Sup $(a + S) = a +$ Sup S.

(b) State Archimedean Property of real numbers. Show that if $y > 0$, then there exists $n \in N$ such that $\frac{1}{2^n} < y$.

(c) Define a countable and an uncountable set. Is the set Q' of irrational numbers uncountable? Justify your answer. (6,6)

(b) Prove that the sequence $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$, $x \in \mathbb{R}$ converges uniformly on $\mathbb{R}$.

(c) Find the upper and lower Darboux integrals for $f(x) = x^3$ on the interval [0, a]. Is f integrable on [0, a]? (6.5,6.5)

6. (a) State Weierstrass M-Test for uniform convergence of series. Hence, show that

$$\sum \frac{1}{x^2+n^2}, \ \forall x \in R$$

is uniformly convergent.

(b) Show that every continuous function defined on [a, b] is Riemann integrable.

(ii) Test for convergence the series :

(1) $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+---.$

(2) $\frac{1}{\log 2}+\frac{1}{(\log 3)^2}+\frac{1}{(\log 4)^3}+\cdots.$

(6.5,6.5)

5. (a) (i) Define an absolutely convergent series. Is every convergent series absolutely convergent? Justify your answer.

(ii) Test for convergence the series:

(1) $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}---$

(2) $\sum_{n=1}^{\infty}(-1)^n.e^{-n}$.

2. (a) Define the convergence of a sequence (x_n) of real numbers. Using the definition, evaluate the following limits :

(i) $\lim_{n\to\infty}\sqrt{n+1}-\sqrt{n}$

(ii) $\lim_{n\to\infty}\left(\frac{(-1)^n n}{n^2+1}\right)$

(b) Prove that $\lim_{n\to\infty} n^{1/n}=1$. Further, evaluate

$\lim_{n\to\infty}(n^{1/n^2})$.

(c) Prove that a convergent sequence of real numbers is bounded. Is the converse true? Justify your answer. (6,6)

3. (a) State Monotone Convergence Theorem. Show that the sequence (x_n) defined by

$$x_1 = 1; x_{n+1} = \sqrt{2 + x_n}, \ \forall n \in N$$ is convergent.

Also, find $\lim_{n \to \infty} x_n$.

(b) Define subsequence of a sequence (x_n) of real numbers.

Prove that if sequence (x_n) of real numbers converges to a real number x, then any subsequence of (x_n) also converges to x.

(c) State Cauchy's Convergence Criterion for sequences of real numbers. Show directly from the definition that the following sequence is a Cauchy sequence:

$$\left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right).$$ (6.5,6.5)

4. (a) State and prove Comparison test for positive term series. Hence, show that the following series converges :

$$1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \cdots.$$

(b) Suppose that (x_n) is a sequence of non-negative real numbers. Prove that the series $\sum x_n$ converges if and only if the sequence $S = (s_k)$ of partial sums is bounded.

(c) (i) State (without proof) D'Alembert's ratio test for an infinite series.

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1351**

Unique Paper Code : **32351402**

Name of the Paper : **Riemann Integration and Series of Functions**

Name of the Course : **B.Sc. (H) Mathematics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) Prove that a bounded function f on $[a, b]$ is integrable if and only if for each $\in > 0$, there exists $\delta > 0$ such that :

$$\text{mesh}(P) < \delta \Rightarrow U(f, P) - L(f, P) < \in,$$

for all partitions P of $[a, b]$. 6

(*b*) Show that if f is integrable on $[a, b]$, then $|f|$ is integrable on $[a, b]$, and

$$\left|\int_a^b f\right| \leq \int_a^b |f|.$$

Hence, show that :

$$\left|\int_{-2\pi}^{2\pi} x^2 \sin^8(e^x)dx\right| \leq \frac{16\pi^3}{3}.$$ 6

(*c*) Show that every continuous function on $[a, b]$ is integrable. Is the converse true ? Justify. 6

2. (a) Let $f(x) = x^2$, $x \in [0, b]$. Calculate upper and lower Darboux integrals for f on $[0, b]$. Is f integrable on $[0, b]$? 6.5

(b) If g is a continuous function on $[a, b]$ that is differentiable on (a, b) and if g' is integrable on $[a, b]$, then prove that :

$$\int_a^b g' = g(b) - g(a). \qquad 6.5$$

(c) State Intermediate value theorem for integrals. Suppose f and g are continuous functions on $[a, b]$ such that $\int_a^b f = \int_a^b g$. Prove that there exists $x \in [a, b]$ such that $f(x) = g(x)$. 6.5

3. (a) Show that the function $\beta : \mathbf{R}^+ \times \mathbf{R}^+ \to \mathbf{R}$ given by :

$$\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$$

is a well-defined function. Find the value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$. 6

(b) Discuss the convergence or divergence of the following improper integrals :

(i) $\int_1^2 \frac{\sqrt{x}}{\log x}\, dx$ 3

(ii) $\int_1^\infty \frac{2x^2}{x^4 - 1}\, dx$. 3

(c) Let f be a function defined on $[a, b]$ such that for each $x \in (a, b)$, f is bounded and integrable on $[a, x]$ and f has an infinite discontinuity at b. Show that if $\int_a^b |f|$ converges, then so does $\int_a^b f$. 6

4. (a) Show that a sequence $\langle f_n \rangle$ of functions on $A \subseteq \mathbf{R}$ converges uniformly to f on A if and only if $f_n - f \ {}_A \to 0$. Deduce that the sequence $\langle f_n \rangle$ where $f_n(x) = x/n$ is uniformly convergent on [0, 1]. 6.5

(b) Let $\langle f_n \rangle$ be a sequence of bounded functions on $A \subseteq \mathbf{R}$ that converges uniformly to a function f on A. Show that the function f is bounded on A. 6.5

(c) Discuss the uniform convergence of the sequence $\langle f_n \rangle$ on [0, 1], where

$$f_n(x) = nxe^{-nx^2}\ \forall x \in [0, 1].$$ 6.5

5. (a) Show that the series of function $\sum_{n=1}^{\infty} \frac{1}{n^2x^2}$ $x \neq 0$ converges uniformly for $|x| \leq a$ for some $a > 0$, but is not uniformly convergent on $\mathbf{R} \setminus \{0\}$. 6.5

(b) State and prove Weierstrass M-test and hence examine the uniform convergence of the series :

$$\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right), \text{ for } x \in [-a, a], \text{ for all } a > 0.$$ 6.5

(c) Prove that the series :

$$\sum_{n=1}^{\infty} \frac{\cos(x^2+1)}{n^2}$$

represents a continuous function on **R**. 6.5

6. (a) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence R > 0 (possibly $R = +\infty$). If there exists a real number R_1, such that $0 < R_1 < R$, then show that the power series $\sum_{n=0}^{\infty} a_n x^n$ is convergent uniformly on $[-R_1, R_1]$. 6

(b) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence R > 0. Then show that the function f is integrable on (–R, R). Also, show that for $x \in (-R, R)$, we have :

$$\int_0^x f(t)\, dt = \sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}.$$ 6

(c) Find the exact interval of convergence of the following power series :

(i) $\sum_{n=1}^{\infty} \frac{x^n}{\ln n}$ 3

(ii) $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$. 3

(c) Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation defined by

$$T(a + bx + cx^2) = (a - c) + (a - c)x + (b - a)x^2 + (c - b)x^3.$$

Find nullspace N(T) and range space R(T). Also verify Rank-Nullity theorem. (6,6,6)

6. (a) Let V and W be finite-dimensional vector spaces having ordered bases β and γ respectively, and let $T: V \to W$ be linear. Then for each $u \in V$, show that

$$[T(u)]_\gamma = [T]_\beta^\gamma [u]_\beta.$$

(b) Let V, W and Z be vector spaces and let $T: V \to W$ and $U: W \to Z$ be linear transformations. Then prove that

(i) If UT is one-to-one, then T is one-to-one.

(ii) If UT is onto, then U is onto.

(c) Let $A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$ and let $\beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ be an ordered basis for $\mathbb{R}^2$. Find $[L_A]_\beta$ and also find an invertible matrix Q such that $[L_A]_\beta = Q^{-1}AQ$. where L_A is a left-multiplication transformation.

(6.5,6.5, 6.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1352 **J**

Unique Paper Code : 32351403

Name of the Paper : Ring Theory and Linear Algebra – I

Name of the Course : **CBCS B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each of the question.

1. (a) Define a nilpotent element in a ring. Show that the nilpotent elements of a commutative ring R form a subring of R.

(b) Give an example of a commutative ring which is not an integral domain. Show that a finite integral domain is a field.

(c) In a commutative ring of characteristic 2, prove that the idempotents form a subring. (6,6,6)

2. (a) Define prime ideal of a commutative ring. Let R be a commutative ring with unity and A be an ideal of R. Then, show that R/A is an integral domain if and only if A is prime ideal of R.

(b) Show that $\dfrac{\mathbb{R}[x]}{(x^2+1)}$ is a field.

(c) Define subring of a ring. Let R be the ring of continuous functions from $\mathbb{R}$ to $\mathbb{R}$. Let $A = \{f \in R : f(0)$ is an even integer$\}$. Show that A is a subring of R, but not an ideal of R.

(6.5,6.5,6.5)

3. (a) Define a ring homomorphism from a ring R to a ring S. Determine all ring homomorphisms from $\mathbb{Z}_6$ to $\mathbb{Z}_{18}$.

(b) Let R be a ring with unity 1. Prove that the mapping $\phi: \mathbb{Z} \to R$ given by $\phi(n) = n.\ 1$ is a ring homomorphism. Hence, show that if R has characteristic $n > 0$, then R contains a subring isomorphic to $\mathbb{Z}_n$.

(c) Let $\mathbb{Z}\left[\sqrt{3}\right]=\left\{a+b\sqrt{3}: a, b \in \mathbb{Z}\right\}$ and $H=\left\{\begin{pmatrix} a & 3b \\ b & a \end{pmatrix}: a, b \in \mathbb{Z}\right\}$. Show that $\mathbb{Z}\left[\sqrt{3}\right]$ and H are isomorphic as rings.

(6,6,6)

4. (a) (i) Let $\mathcal{C}$ be a collection of subspaces of a vector space V and let W denote the intersection of subspaces in $\mathcal{C}$. Prove that VP is a subspace of V.

(ii) Show that $W = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : 2a_1 + a_2 + a_3 = 0\}$ is a subspace of $\mathbb{R}^3$.

(b) (i) Let S_1 and S_2 are arbitrary subsets of a vector space V. If $S_1 \subseteq S_2$, then show that span $(S_1) \subseteq$ span (S_2).

(ii) Check whether the vectors (1,0,0), (1,0,1) and (1,1,1) generate $\mathbb{R}^3$.

(c) Define a linearly independent subset of a vector space V. Let 5 be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Prove that $S \cup \{v\}$ is linearly dependent if and only if $v \in$ span (S).

(6.5,6.5,6.5)

5. (a) Let V and W be vector spaces and $T: V \to W$ be a linear transformation. Then prove that the null space N(T) and the range space R(T) are subspaces off and W, respectively.

(b) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x_1, x_2) = (x_1 - x_2, x_1, 2x_1 + x_2)$. Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma = \{(1,1,0), (0,1,1), (2,2,3)\}$ be an ordered basis for $\mathbb{R}^3$. Compute $[T]_\beta^\gamma$.

SL No of QP : **5534**
Unique Paper Code : 2352012401

Name of the Paper : Sequences and Series of Functions

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. **All** questions carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) Define uniform convergence for sequence of functions (f_n) defined on $\mathbb{R}$. Show that a sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $||f_n - f||_A \to 0$.

   (b) Show that the sequence $f_n(x) = \frac{nx}{1+n^2x^2}$ converges uniformly in the interval $[a, \infty)$ where $a > 0$ but it does not converge uniformly on the interval $[0, \infty)$.

   (c) Let $(f_n), (g_n)$ be sequences of bounded functions on A that converge uniformly on A to f, g, respectively. Show that $(f_n g_n)$ converges uniformly on A to fg.

2. (a) Let $f_n(x): [0,1] \to \mathbb{R}$ be defined for $n \geq 2$ by

$$f_n(x) := \begin{cases} n^2 x & for\ 0 \leq x \leq \frac{1}{n} \\ -n^2\left(x - \frac{2}{n}\right) & for\ \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & for\ \frac{2}{n} \leq x \leq 1 \end{cases}$$

   Evaluate pointwise limit f of f_n and show that (f_n) does not converge uniformly to f on the interval $[0,1]$.

   (b) Let (f_n) be a sequence of integrable functions on $[a, b]$ and suppose that (f_n) converges uniformly to f on $[a, b]$. Show that f is integrable on $[a, b]$ and

$$\int_a^b f = \lim_{n\to\infty} \int_a^b f_n.$$

   (c) Show that the sequence $f_n(x) = n^2x^2e^{-nx}$ converges uniformly on $[a, \infty)$ where $a > 0$ but does not converge uniformly on the interval $[0, \infty)$.

3. (a) State and prove Weierstrass M-test for uniform convergence of series of functions. Hence, check for uniform convergence of series $\sum_{n=1}^{\infty}\frac{1}{x^2+n^2}$, $x \in \mathbb{R}$.

(b) Discuss the pointwise convergence of the series $\sum_{n=1}^{\infty}\frac{x^n}{x^n+1}$, $x \geq 0$. Also, show that the given series is not uniformly convergent on $[0,1)$ but is uniformly convergent on $[0,\frac{1}{2})$.

(c) Prove that if $\sum f_n$ converges uniformly to f on a domain $D \subseteq \mathbb{R}$ and each f_n is continuous on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ then f is continuous on D.

4. (a) Let f_n be a real valued function on $[a,b]$ that has derivative f_n' on $[a,b]$ for each $n \in \mathbb{N}$. Suppose that the series $\sum f_n$ converges for at least one point of $[a,b]$ and the series of derivatives $\sum f_n'$ converges uniformly on $[a,b]$. Show that there exists a real valued function f on $[a,b]$ such that $\sum f_n$ converges uniformly to f on $[a,b]$. Also, that f has a derivative on $[a,b]$ and $f' = \sum f_n'$.

(b) State and prove the Cauchy criterion for the uniform convergence of series of functions $\sum f_n$. Use it to prove the non uniform convergence of $\sum_{n=1}^{\infty}\frac{1}{n^2x^2}$ for $|x| < 1$.

(c) Prove that the series $\sum_{n=1}^{\infty}\frac{1}{x^n+1}$ is convergent for $x > 1$ and divergent for $0 \leq x \leq 1$. Also, show that the series converges uniformly on $[a,\infty)$, $a > 1$.

5. (a) (i) Find the exact interval of convergence for the power series $\sum_{n=1}^{\infty} a_n x^n$ where $a_n = \frac{3^n}{n4^n}$.

(ii) Let $f(x) = |x|$ for $x \in \mathbb{R}$. Is there a power series $\sum_{n=0}^{\infty} a_n x^n$ such that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ for all x ? Justify.

(b) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $R > 0$. If $0 < R_1 < R$, then show that the power series converges uniformly on $[-R_1, R_1]$ to a continuous function.

(c) Show that $\arctan x = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}x^{2n+1}$ for $|x| < 1$. Apply Abel's theorem to show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

6. (a) Define logarithm function L on $(0,\infty)$. Show that the function L satisfies the following:

(i) $L'(x) = \frac{1}{x}$ for $x > 0$.

(ii) $L(xy) = L(x) + L(y)$ for $x > 0, y > 0$.

(b) Show that $\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}$ for $|x| < 1$. Evaluate $\sum_{n=1}^{\infty}\frac{n}{3^n}$.

(c) Show that there does not exist a sequence of polynomials converging uniformly on $\mathbb{R}$ to $f(x) = e^x$.

(c) Prove that in a ring R with unity 1,

(i) if 1 has infinite order under addition, then the characteristic of R is 0,

(ii) if 1 has finite order n under addition, then the characteristic of R is n.

Hence or otherwise state the characteristics of the rings $\mathbb{Z}$, $M_2(\mathbb{Z})$ and $\mathbb{Z}_3[i]$.

6. (a) State the ideal test. Let R be a commutative ring with unity and let $a \in R$. Then verify that the sets $I = \{ra \mid r \in R]$ and $S = \{ar \mid r \in R)$ are ideals of R.

(b) Let $\phi: R \to S$ be a ring homomorphism. Prove that :

(i) $\phi(nr) = n\phi(r)$ and $\phi(r^n) = (\phi(r))^n$ for any $r \in R$ and any positive integer n.

(ii) $\phi(R)$ is commutative if R is commutative.

(c) Prove that every ring homomorphism ϕ from $\mathbb{Z}_n$ to itself has the form $\phi(x) = ax$, where $a^2 = a$. Also, find $\text{Ker}\phi$.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5106 **J**

Unique Paper Code : 2354000009

Name of the Paper : Abstract Algebra

Name of the Course : **COMMON PROG GROUP**

Semester : VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Each part carries **7.5** marks.
5. Use of Calculator not allowed.

1. (a) Define SL(2, $\mathbb{F}$) and find the inverse of the element $\begin{bmatrix} 3 & 4 \\ 4 & 4 \end{bmatrix}$ in SL(2, $\mathbb{Z}_5$).

(b) Describe the symmetries of a square. Construct the corresponding Cayley Table.

(c) Prove that the set $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \middle| \, a, b, c \in \mathbb{R} \right\}$ is a group under matrix multiplication.

2. (a) If H and K are subgroups of a group G, show that $H \cap K$ is a subgroup of G. What can you say about $H \cup K$? Justify.

(b) Let G be a group. Then prove that: $c(a) = c(a^{-1})$ $\forall$ a $\in$ G, where c(a) is the centralizer of α in G.

(c) Prove that the center of a group G is a subgroup of G. Find the center of the group D_4. Justify.

3. (a) Define a cyclic group. Find an example of a noncyclic group, all of whose proper subgroups are cyclic. Justify your answer.

(b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{pmatrix}$,

$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$.

Write α, β and $\alpha\beta$ as product of disjoint cycle and also find β^{-1}.

(c) State the Lagrange's Theorem. Let $|G| = 60$, then what are the possible orders for subgroups of G.

4. (a) Let H be a subgroup of a group G, then show that either $\alpha H = bH$ or $\alpha H \cap bH = \phi$ for every α, b $\in$ G. Also, find all the left cosets of {1,11} in U(30).

(b) Define the factor group. Also, let $G = \mathbb{Z}_{24}$, then find the order of the element 14+<8> in the factor group $\mathbb{Z}_{24}$/<8>.

(c) Consider the map f: $\mathbb{Z} \to \mathbb{Z}_n$ defined by f(m) = mmodn. Show that f is a homomorphism. Also, find the Kerf.

5. (a) Show that the set $\mathbb{Z}[x]$ of all polynomials in the variable x with integer coefficients under addition and multiplication is a commutative ring with unity f(x) = 1.

(b) State the subring test. Prove that in a ring R, the set $\{x \in R \mid \alpha x = x\alpha \text{ for all } \alpha \in R\}$ is a subring of R. Find all the subrings of the ring $\mathbb{Z}$.

(b) Let G be a group of order 12. Prove that either G has a normal Sylow 3-subgroup or G is isomorphic to alternating group A_4.

(c) State Sylow's first theorem. Let G be a group of order 77. Prove that G has a normal Sylow 7-subgroup and a normal Sylow 11-subgroup. Further, prove that G is cyclic.

5. (a) State and prove the Index theorem. Use this theorem to show that there is no simple group of order 216.

(b) Let G be a finite group, and let p be the smallest prime dividing the order of G. Prove that if G has a subgroup of index p, then it must be a normal subgroup of G. Is it always true that G has a subgroup of index p? Justify.

(c) Prove that no simple group has order pqr, where p, q and r are distinct primes.

6. (a) Prove that a group G is solvable if and only if $G^{(n)} = \{e\}$ for some positive integer n, where $G^{(n)}$ is the n^{th} derived subgroup of G.

(b) Find a composition series for the symmetric group S_4.

(c) Prove that any finite p-group is nilpotent.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 5514 **J**

Unique Paper Code	:	2352013601
Name of the Paper	:	Advanced Group Theory
Name of the Course	:	**B.Sc. (H) Mathematics**
Semester	:	VI – DSC-16

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and are of **15** marks each.
3. Attempt any **five** parts from Question **1**. Each part is of **3** marks.
4. Attempt any **two** parts from each of the Questions **2** to **6**. Each part is of **7.5** marks.

1. (i) Prove that the left regular action of a group G on itself is a faithful action.

(ii) Prove that the kernel of an action of a group G on a set A is a normal subgroup of G.

(iii) Let σ be an m-cycle in the symmetric group S_n. Find $|C_{S_n}(\sigma)|$, the order of centralizer of σ.

(iv) Is a group of order 210 simple? Justify.

(v) Define commutator subgroup G' of a group G. Prove that G' is normal in G.

(vi) Prove that the center of a group is a characteristic subgroup.

(vii) Define a solvable group. Prove that every abelian group is solvable.

2. (a) For a group G, consider the mapping $G \times G \to G$ given by $g.a = gag^{-1}$. Prove that this defines a group action of G on itself. Also, find the kernel of this action and the stabilizer G_x of an element $x \in G$.

(b) Let $G = D_8$ be the dihedral group of order 8 and let $A = \{1, r, r^2, r^3\}$, where r denotes a clockwise rotation of the square by $\frac{\pi}{2}$ radians. Show that the normalizer $N_{D_8}(A) = D_8$ and the centralizer $C_{D_8}(A) = A$.

(c) Let p be a prime number and let G be a group of order p^{α} for some $\alpha \geq 1$. Prove that G has a nontrivial center Z(G). Deduce that every group of order p^2 is abelian.

3. (a) Let the symmetric group S_n act on the set $A = \{1, \ldots\ldots, n\}$ by $\alpha.a = \alpha(a)$, $\forall\ \alpha \in S_n$ $a \in A$. Prove that this action is transitive.

(b) Let the dihedral group D_8 act via its natural action on the set $A = \{1,2,3,4\}$ consisting of four vertices of a square. Label these vertices 1,2,3,4 in a clockwise direction. Let r be the rotation of the square clockwise by $\frac{\pi}{2}$ radians and s be the reflection in the line which passes through vertices 1 and 3. Find the stabilizer of all the four vertices of square.

(c) Let G be a finite group and let $g_1, g_2, \ldots\ldots g_r$ be representatives of the distinct conjugacy classes of G not contained in the centre Z(G) of G. Then prove the class equation:

$|G| = |Z(G)| + \sum_{i=1}^{r} |G : C_G(g_i)|$, where $C_G(g_i)$ is the centralizer of the element g_i in G. Verify the class equation for the symmetric group S_3.

4. (a) Let G be a finite group and p be a prime dividing the order of G. Let P be a Sylow p-subgroup of G, and let n_p denote the number of Sylow p-subgroups of G. Assume that G acts transitively on the set of its Sylow p-subgroups by conjugation.

(i) Describe the orbit of P under this action.

(ii) Show that $n_p = |G : N_G(P)|$, where $N_G(P)$ denotes the normalizer of P in G.

P.T.O.

(ii) Let T be a linear operator on a finite dimensional complex inner product space V. Show that T is unitary if and only if T is normal and $|\lambda| = 1$ for every eigen value λ of T. (2+5.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5554 **J**

Unique Paper Code	:	2352013602
Name of the Paper	:	Advanced Linear Algebra
Name of the Course	:	**Bachelor of Science (Honours Course) Mathematics**
Semester	:	VI

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **TWO** parts from each Question.

1. (a) Let T be a linear operator on R^2 defined as

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}.$$

For the ordered basis $\beta = \left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\}$ and

$\beta' = \left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$, of R^2, find the change of

coordinate matrix Q that changes β'-coordinates into β-coordinates. Also, verify that

$[T]_{\beta'} = Q^{-1}[T]_{\beta}Q.$ (2.5+5)

(b) Let $V = P_1(R)$ and for $p(x) \in V$, let $f_1, f_2 \in V^*$ be defined as

$$f_1(p(x)) = \int_0^1 p(t)\,dt$$

(c) Find the best fit linear function for the data $\{(-3,9), (-2,6), (0,2), (1,1)\}$ using the least squares approximation. Also, compute the error E.

(5+2.5)

6. (a) Let T be a normal operator defined on a finite dimensional real inner product space V whose characteristic polynomial splits. Prove that V has an orthonormal basis of eigen vectors of T. Hence prove that T is self-adjoint.

(5+2.5)

(b) For the following matrix A, find an orthogonal matrix P and a diagonal matrix D such that $P^tAP = D$.

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$ (7.5)

(c) (i) State the Spectral theorem.

(c) Let $W = \text{span}(\{(i, 0, 1)\})$ in C^3. Find the orthonormal bases for W and $W^{\perp}$. (7.5)

5. (a) Let $V = P(R)$ with the inner product defined as

$$\langle f(x), g(x)\rangle = \int_0^1 f(t)\, g(t)\,dt,\ \forall\, f(x),\ g(x) \in V.$$

Find the orthogonal projection of the vector $h(x) = 4 + 3x - 2x^2$ on the subspace $W = P_1(R)$. (7.5)

(b) (i) Let V be a finite dimensional inner product space and β be an orthonormal basis for V. If T is a linear operator on V, show that

$$[T^*]_\beta = ([T]_\beta)^*.$$

(ii) For the inner product space $V = C^2$ and linear operator

$$T(z_1, z_2) = (2z_1 + iz_2, (1-i)z_1),$$

evaluate T^* at $z = (3-i,\ 1+2i)$. (4.5+3)

and $f_2(p(x)) = \int_0^2 p(t)\,dt$.

Prove that $\{f_1, f_2\}$ is a basis for V^* and find a basis of V for which it is the dual basis. (3+4.5)

(c) Let V be finite dimensional vector space. Define the annihilator S^0 of a subset S of V and prove that S^0 is a subspace of V^*. If W_1 and W_2 are subspaces of V, prove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0. \quad (3.5+4)$$

2. (a) Let $A = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix} \in M_{2\times 2}(C)$. Determine all eigen values of A and for each eigen value λ of A, find the set of eigen vectors corresponding to λ. Also, find a basis for C^2 consisting of eigenvectors of A. (2.5+5)

(b) Let T be a linear operator on $P_2(R)$ defined as

$$T(f(x)) = f(0) + f(1)(x + x^2).$$

Test the linear operator T for diagonalizability. If T is diagonalizable, then find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (2.5+5)

(c) Let T be a diagonalizable linear operator on a finite dimensional vector space V and $\lambda_1, \lambda_2, \ldots \ldots \lambda_k$ be the distinct eigen values of T. Prove that

$V = E_{\lambda_1} \oplus E_{\lambda_2} \ldots\ldots \oplus E_{\lambda_k}$, where E_{λ_i} is the eigen space of λ_i, for all i. (7.5)

3. (a) Let T be a linear operator on the vector space $V = R^4$ defined as

$$T(a, b, c, d) = (a+b,\ b-c,\ a+c,\ a+d)$$

Find an ordered basis of the T-cyclic subspace W of V generated by $z = e_1$. Also, find the characteristic polynomial of T_W. (3+4.5)

(b) Let T be a linear operator defined on a finite dimensional vector space V. Prove that the characteristic polynomial and the minimal polynomial of T have the same zeros. (7.5)

(c) Let T be a linear operator on $V = M_{n\times n}(R)$ defined as $T(A) = A^t$. Find the minimal polynomial of T. Hence show that T is diagonalizable. (7.5)

4. (a) Let V be an inner product space, prove that the following inequality holds

$$|\langle x, y\rangle| \le \|x\|\,\|y\|, \text{ for all } x, y \in V.$$

Also, verify that the inequality holds for $x = (1, 2i, 1 + i)$, $y = (5 + i, 1, 2)$ in C^3. (5+2.5)

(b) Let V be an inner product space and let S be an orthogonal subset of V consisting of nonzero vectors. Prove that S is linearly independent. (7.5)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **5574**

Unique Paper Code : **2352013603**

Name of the Paper : **Complex Analysis**

Name of the Course : **B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)**

Semester : **VI**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and carry equal marks.

Each question contains three parts (*a*), (*b*) and (*c*), attempt only *two* parts.

1. (*a*) (*i*) Sketch the set $S = \left\{z \in \mathbf{C} : |z| \le 1, -\frac{\pi}{2} \le \arg z \le \frac{\pi}{2}\right\}$. 3

(*ii*) Under the map $f(z) = z^2$, draw the image of the set : 4.5

$$S = \left\{z \in \mathbf{C} : |z| = 1, -\frac{\pi}{4} \le \arg z \le \frac{\pi}{4}\right\}.$$

(*b*) Under the map $f(z) = e^z$ draw the image of the rectangular region

$$R = \left\{z = x + iy \in \mathbf{C} : 1 \le x \le 2, -\frac{\pi}{4} \le y \le \frac{\pi}{2}\right\}.$$ 7.5

P.T.O.

(c) Does $\lim_{z \to 0} \left(\frac{z}{\overline{z}}\right)^2$ exist ? Justify your answer. 7.5

2. (a) Show that the function $f(z) = \overline{z}$ is nowhere differentiable on **C**. 7.5

(b) (i) Giving proper justification, write two differences between the functions : 3.5

$$f(x) = e^x,\ x \in \mathbf{R} \text{ and } f(z) = e^z,\ z \in \mathbf{C}$$

(ii) Solve the equation $e^z = -2$. 4

(c) Find out the values of $\sin^{-1} 2$. 7.5

3. (a) (i) Let a function f be integrable along a contour C. Prove that : 3.5

$$\int_{-C} f(z)dz = -\int_{C} f(z)dz.$$

(ii) Define a simple arc. Give an example to explain that a same set of points may represent different arcs. 4

(b) Let C be any simple closed contour, described in the positive sense in the z plane, then compute the value of the integral :

$$\int_C \frac{s^3 + 2s}{(s - z_0)^3}\, ds,$$

(i) when z_0 is inside C, and

(ii) when z_0 is outside C. 7.5

(c) (i) Let $w(t)$ be a piecewise continuous complex valued function defined on an interval $a \le t \le b$, then show that : 3.5

$$\left|\int_a^b w(t)\, dt\right| \le \int_a^b |w(t)|\, dt.$$

(*ii*) Let C be the arc of the circle $|z| = 2$ such that $0 \leq ar\,gz \leq \dfrac{\pi}{2}$. Show that : 4

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}.$$

4. (*a*) State and prove Cauchy integral formula. 7.5

(*b*) Let f be an entire function such that $|f(z)| \leq A|z|$ for all z, where A is a fixed positive number. Show that $f(z) = bz$, where b is a complex constant. 7.5

(*c*) (*i*) Evaluate the following integral along the line segment, say L, from 0 to $\pi + 2i$: 3.5

$$\int_L \cos(z/2) dz.$$

(*ii*) Evaluate the following integral :

$$\int_C \frac{z\,dz}{(9-z^2)(z+i)},$$

where C is the positively oriented circle $|z| = 2$. 4

5. (*a*) State Laurent's series theorem. Give two series expansions in powers of z for the function $f(z) = \dfrac{z+1}{z-1}$ specifying the regions in which those expansions are valid. 7.5

(*b*) If a power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges when $z = z_1$ $(z_1 \neq z_0)$, then prove that it is absolutely convergent at each point z in the open disk $|z - z_0| < R_1$ where $R_1 = |z_1 - z_0|$. 7.5

(c) Define residue. Evaluate the residue of the function $f(z) = \dfrac{e^z - 1}{z^4}$ and use it to find the integral $\int_C \dfrac{e^z - 1}{z^4} dz$, where C is the positively oriented unit circle $|z| = 1$. 7.5

6. (a) Define an isolated singular point. For the below given functions, find the isolated singular point and determine whether it is a removable singular point, an essential singular point, or a pole : 7.5

(i) $\dfrac{\sin z}{z}$ and

(ii) $\dfrac{e^{2z}}{(z-1)^2}$.

(b) Let z_0 be an isolated singular point of a function f. Then show that the following two statements are equivalent :

(i) z_0 is a pole of order $m (m = 1, 2, ...)$ of f;

(ii) $f(z)$ can be written in the form $f(z) = \dfrac{\varphi(z)}{(z - z_0)^m}$, where $\varphi(z)$ is analytic and non-zero at z_0.

Further, establish that

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{\varphi^{(m-1)}(z_0)}{(m-1)!} \text{ when } m = 1, 2, 3, ...$$

where $\varphi^{(0)} \equiv \varphi$. 7.5

(c) Using residues, evaluate the following integral : 7.5

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} d\theta.$$

(c) Determine the residue of the below given complex functions at z = 0

(i) $\dfrac{\sin hz}{z^4\left(1-z^2\right)}$

(ii) $\dfrac{\cot z}{z^4}$ (3+3=6)

(d) Define singular point and isolated singular point of a complex function. Consider the function

$$f(z)=\frac{\sin z}{(z-1)^3(z+2)}$$

Identify all isolated singularities of the function f(z). (3+3=6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1353 **J**

Unique Paper Code : 32351601

Name of the Paper : Complex Analysis

Name of the Course : **B.Sc. (H) Mathematics CBCS-LOCF**

Semester : VI – Core

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Use of Calculator is not allowed.

1. (a) Find and sketch the image of each branch of the hyperbola $x^2 - y^2 = c$, $c > 0$ under the transformation $w = z^2$. (6)

(b) Prove that (i) $\lim\limits_{z\to 0}\dfrac{z^{-2}}{z} = 0$ (ii) $\lim\limits_{z\to\infty}\dfrac{z^2+1}{z-1} = \infty$

(3+3)

(c) Prove that if $f'(z) = 0$ everywhere in a domain D, then f(z) must be constant throughout D. (6)

(d) Show that the function $f(z) = \cosh x \cos y + i \sinh x \sin y$ is entire. (6)

2. (a) (i) Find all values of z such that $e^z = -2$.

(ii) Show that if e^z is real, then $\operatorname{Im} z = n\pi$, $n \in \mathbb{Z}$.

(3+3.5)

(b) Show that $\sin z = 0$ if and only if $z = n\pi$, $n \in \mathbb{Z}$.

(6.5)

(c) If a series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges to f(z) at all points interior to some circle $|z - z_0| = R$, then prove that it is the Taylor series for the function f(z) in powers of $z - z_0$. (6.5)

(d) Prove that when

$$z \neq 0,\ \frac{\sin\left(z^2\right)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^{10}}{7!} + \cdots \qquad (6.5)$$

6. (a) Prove that if a sequence of complex numbers converges, then its n^{th} term converges to zero as $n \to \infty$. is the converse true? Justify. (6)

(b) Derive the Laurent series expansion for the function $f(z) = e^{\frac{1}{z}}$ and hence show that $\int_\Gamma e^{\frac{1}{z}}\,dz = 2\pi i$, where Γ is any positively oriented simple closed contour around the origin.

(3+3=6)

(ii) $\int_C \frac{z^2}{z+3}\,dz$

where C is the unit circle $|z| = 1$ in either direction.

(1.5+2.5+2.5=6.5)

(c) Show that if f is analytic at a given point then its derivatives of all higher orders are analytic there too. (6.5)

(d) State and prove fundamental theorem of algebra. (6.5)

5. (a) Suppose that the sequence (z_n), where $z_n = x_n + iy_n$, $n = 1,2,\ldots$ and $z = x + iy$ then prove that $\lim_{n\to\infty} z_n = z \Leftrightarrow \lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, and hence check the convergence behavior of complex sequence $z_n = \frac{2}{n^3} + 5i$, $(n = 1,2,\ldots)$.

(3.5+3=6.5)

(b) Find Maclaurin series representation of $f(z) = \cosh z$. (6.5)

(c) Show that

(i) $\text{Log}\,[(1+i)^2] = 2\ \text{Log}\,(1+i)$

(ii) $\text{Log}\,[(-1+i)^2] \neq 2\ \text{Log}\,(-1+i)$.

(3+3.5)

(d) Show that

(i) $\log e = 1 + 2n\pi i,\ n \in \mathbb{Z}$.

(ii) $\log i = \left(2n + \frac{1}{2}\right)\pi i,\ n \in \mathbb{Z}$. (3+3.5)

3. (a) Let $w(t) = u(t) + i\,v(t)$ be a complex-valued function of a real parameter t.

(i) Define the definite integral of $w(t)$ over $a \le t \le b$ and evaluate

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt.$$

(ii) Show that if w(t) denotes a continuous function on [–a, a] which is even; that is, $w(-t) = w(t)$ for each point t in [–a, a], then

$$\int_{-a}^{a} w(t)\,dt = 2\int_{0}^{a} w(t)\,dt.$$ (3+3)

(b) What do you mean by differentiable arc and its length? Show that the length of an arc C remain invariant under change in the representation for C. (2+4=6)

(c) Let $f(z) = \pi \exp(\pi \bar{z})$ and C be the boundary of the square with vertices at the points 0,1,1 + i and i, the orientation of C being the counter clockwise direction. Using parametric representation of C, evaluate $\int_C f(z)\,dz$. (6)

(d) Without evaluating the integral, show that

$$\left|\int_C \frac{z+4}{z^3-1}\,dz\right| \le \frac{6\pi}{7}$$

where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. (6)

4. (a) Use the method of antiderivatives to show that for every contour C extending from a point z_1 to z_2,

$$\int_C z^n dz = \frac{z_2^{n+1} - z_1^{n+1}}{n+1} \quad \text{for each } n = 0,1,2,\ldots$$

State the corresponding result used. (4.5+2=6.5)

(b) State Cauchy Goursat's theorem and hence evaluate

(i) $\int_C \mathrm{Log}(z+2)\,dz$

(b) Find the optimal strategies for both player A and B and value of the game whose payoff matrix is given below.

	B_1	B_2
A_1	1	-3
A_2	3	5
A_3	-1	6
A_4	4	1
A_5	2	2
A_6	-5	0

(c) Solve the following system of equations by using Simplex method :

$$x_1 + x_2 = 1$$
$$2x_1 + x_2 = 3.$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1360 **J**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Define a convex set. Prove that the set S defined as;

$$S = \{(x_1, x_2): x_1^2 + x_2^2 \le 1\} \text{ is a convex set.}$$

(b) Sketch the feasible region of the given linear programming problem and hence find its optimal solution

$$\begin{aligned} \text{Max } Z &= 3x_1 + x_2 \\ \text{s.t,} \quad -x_1 + 2x_2 &\leq 0 \\ x_2 &\leq 4 \\ x_1, x_2 &\geq 0. \end{aligned}$$

(c) Reduce $2a_1 + 3a_2 + a_3 = b$ to a Basic Feasible Solution, if

$a_1 = [2,3]$, $a_2 = [1,1]$, $a_3 = [4,5]$ and $b = [11,14]$.

Name the basic and non-basic variables in the reduced Basic Feasible Solution.

2. (a) Apply Two-phase method to find the solution of the following linear programming Problem :

$$\begin{aligned} \text{Min } Z &= 5x_1 + 11x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\leq 4 \\ 3x_1 + 4x_2 &\geq 24 \\ 2x_1 - 3x_2 &\geq 6 \\ x_1, x_2 &\geq 0. \end{aligned}$$

	Depots				
Factories	D_1	D_2	D_3	D_4	Capacity (tons)
R_1	5	7	13	10	700
R_2	8	6	14	13	400
R_3	12	10	9	11	800
Requirements	200	600	700	400	

Determine the optimum distribution for this company to minimize the shipping cost using North-West Comer Rule, Least Cost Method and Vogel's Approximation Method. Compare the solutions (in terms of costs).

5. (a) Define saddle point. Solve the game whose payoff matrix is given below.

	Player B			
Player A	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

(b) A company wish to hire five employees for five positions claim that they have the required knowledge for each position. Each position can be assigned to one and only one employee. The cost of assignment (per lakh CTC) of position to each employee is given below in the table. Allocate the positions to appropriate employee for optimal assignment.

Employee \ Positions	A	B	C	D	E
I	3	8	2	10	3
II	8	7	2	9	7
III	6	4	2	7	5
IV	8	4	2	3	5
V	9	10	6	9	10

(c) An Oil company has three refineries and four depots. The transportation cost per ton, capacity and requirements are given below in the table.

(b) Solve the following linear programming problem by Simplex method :

$$\begin{aligned} \text{Max } Z &= 3x_1 + 2x_2 \\ \text{s.t, } \quad x_1 &\le 4 \\ x_2 &\le 6 \\ x_1 + x_2 &\le 5 \\ x_1, x_2 &\ge 0. \end{aligned}$$

(c) Using Chame's M-technique to solve the following linear programming problem:

$$\begin{aligned} \text{Max } Z &= x_1 + 2x_2 + 3x_3 \\ \text{s.t, } \quad x_1 + x_2 + x_3 &= 5 \\ 2x_2 + x_3 &\ge 2 \\ 2x_1 + x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0. \end{aligned}$$

3. (a) Find the dual of the following linear programming problem:

$$\begin{aligned} \text{Max } Z &= 5x_1 + 6x_2 \\ \text{s.t, } \quad x_1 + 2x_2 &= 5 \\ -x_1 + 5x_2 &\ge 3 \\ 4x_1 + 7x_2 &\le 8 \\ x_1 \text{ unrestricted}, \; x_2 &\ge 0. \end{aligned}$$

(b) Solve graphically the dual of given primal linear programming problem, then use complementary slackness condition to determine the solution of the primal problem.

$$\begin{aligned} \text{Min } Z &= 2x_1 + x_2 + 3x_3 + 6x_4 \\ \text{s.t.} \quad & x_1 + x_2 + 3x_3 + 2x_4 \geq 3 \\ & 2x_1 + x_2 + x_3 + 3x_4 \geq 2 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

(c) Let x_0 be the feasible solution to the primal problem.

$$\begin{aligned} \text{Max } Z &= CX \\ \text{s.t.} \quad & AX \leq b \\ & X \geq 0. \end{aligned}$$

Where, $X^T, C \in \mathbb{R}^n$, $b^T \in \mathbb{R}^m$ and A = m × n matrix. If w_0 be a feasible solution to the dual of the primal problem.

$$\begin{aligned} \text{Min } Z^* &= b^T w \\ \text{s.t.} \quad & A^T w \geq C^T \\ & w \geq 0. \end{aligned}$$

Where $w^T \in \mathbb{R}^m$. Then prove that $Cx_0 \leq b^T w_0$.

4. (a) A company has three production houses and supplying the product to five warehouses. The cost of production varies from production houses to production houses and cost of transportation from production houses to warehouses also varies. Each Production houses has a certain production capacity and each warehouses have certain amount of requirement. The cost of transportation per unit is given below.

	Warehouse					
Production House	I	II	III	IV	V	Supply
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	

Determine the optimum quantity to be supplied from each production houses to different warehouses at minimum cost.

(c) Find the payoff from a bear spread created using put options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1358 **J**

Unique Paper Code : 32357614

Name of the Paper : MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (H) Mathematics (CBCS-LOCF)**

Semester : VI – DSE

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator/scientific calculator and normal distribution table is allowed.

1. (a) A bank quotes an interest rate of 14% per annum with quarterly compounding. What is the equivalent rate with

   (i) continuous compounding and

   (ii) annual compounding?

   (b) Explain the following type of interest rates :

   (i) Treasury Rates

   (ii) LIBOR

   (iii) Repo Rates

   (c) Portfolio A consists of 1-year zero coupon with a face value of ₹2000 and a 10-year zero coupon bond with face value of ₹6000. Portfolio B consists of a 5.95-year zero coupon bond with face value of ₹5000. The current yield on all bonds is 10% per annum.

   (i) Show that both portfolios have the same duration.

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

   (b) Companies X and Y have been offered the following rates per annum on a $5 million 10-year investment :

	Fix Rate	Floating Rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

   Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T,\ \sigma^2 T\right],$$ where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(K > S_T)$, the probability that a European put option be exercised in a risk-neutral world.

(b) Calculate the price of a five-month European call option on a non-dividend-paying stock with a strike price of ₹60 when the current stock price is ₹62, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum.

(ln(62/60) = 0.0328)

(c) Show that the Black-Scholes-Merton formulas for call and put options satisfy the put- call parity.

(ii) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(You can use the exponential values: e^x = 0.905, 0.368, 0.552, 0.861, 0.223 and 0.409 for x = –0.1, –1.0, –0.595, –0.15, –1.5 and –0.893 respectively)

2. (a) Explain Hedging. How is the risk managed when Hedging is done using?

(i) Forward Contracts

(ii) Options

(b) (i) What is the difference between the over-the counter market and the exchange-traded market?

(ii) An investor enters into a short forward contract to sell 100,000 British pounds for US dollars at an exchange rate of 1.9000 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.8900 and (b) 1.9200?

(c) Write a short note on European call options. Explain the payoffs in different types of call option positions with the help of diagrams.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the vega of a European call and the vega of a European put on a non-dividend-paying stock.

(c) What are the lower and upper bounds for European calls on a non-dividend-paying stock? Calculate the lower bound for the price of a 4-month call option on a non-dividend-paying stock when the stock price is \$28, the strike price is \$25, and the risk-free interest rate is 8% per annum?

4. (a) Consider the standard one-period binomial model where the stock price goes from S_0 to S_0u or S_0d with $d < 1 < u$, and consider an option which pays f_u or f_d in each case, and assume that the interest rate is r and time to maturity is T.

Derive the formula for the no-arbitrage price of the option.

(b) A stock price is currently ₹50. It is known that at the end of six months it will be either ₹45 or ₹55. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹50? (You can use exponential value: $e^{-0.05} = 0.95123$).

(c) A stock price is currently ₹100. Over each of the next two three-month periods it is expected to go up by 8% or down by 7%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month American put option with a strike price of ₹102? (You can use exponential value: $e^{-0.0125} = 0.9876$).

(c) Given the IVP $\frac{dy}{dx} = \frac{e^x}{y}$, $y(0) = 1$, $0 \leq x \leq 1$, find $y(0.5)$ and $y(0.75)$ using Euler's Method. Also find the absolute error at each step size if the exact solution of the given IVP is $y = \sqrt{2e^x - 1}$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2682 **J**

Unique Paper Code : 42357618

Name of the Paper : DSE – Numerical Methods

Name of the Course : **B.Sc. (Prog.) Physical Sciences / Mathematical Sciences (CBCS-LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Round off the number 65468 to four significant digits and then calculate the absolute, relative and percentage errors.

(b) Using Secant method, find the smallest positive root of the equation $x^3 - 9x = -2$ correct to three decimal places.

(c) Determine the number of significant digits in the following numbers

(i) 27.00

(ii) 0.004030

(iii) 1.0056

(iv) 1.02×10^{-4}

(c) The velocity of a particle which starts from rest is given by the following table

t(sec)	0	2	4	6	8	10	12	14	16	18
v(ft/sec)	0	12	16	26	40	44	25	12	5	0

Evaluate using trapezoidal rule the total distance travelled in 18 seconds.

6. (a) Find the approximate value of $\int_0^1 \frac{dx}{x^2+6x+10}$ using Simpson's Rule with 2 and 4 subintervals.

(b) Using Mid Point Method, solve the following initial value problem $\frac{dy}{dx} = x + \sqrt{y}$, $y(0) = 1$, $0 \le x \le 0.4$ by using $h = 0.2$.

(ii) $\delta = E^{1/2} - E^{-1/2}$

μ is average operator, δ is central operator and E is shift operator.

5. (a) Find Richardson's extrapolation of $f(x) = e^x - \sin x$ at $x = 1$, $h = 0.5$, $h = 0.25$ using the central divided difference formula

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}.$$

(b) Use the formula

$$f'(x_i) = \frac{-f(x_i + 2h) + 4f(x_i + h) - 3f(x_i)}{2h}$$

to approximate the derivative of $f(x) = \cos x$ at $x_i = \pi$ taking $h = 0.1$ and $h = 0.01$

2. (a) Perform five iterations of the bisection method to obtain the smallest positive root of the equation $x^3 - 4x + 1 = 0$.

(b) Using Regula falsi method, compute the real root of the equation $x^2 = 2$ correct to four decimal places.

(c) Using Newton-Raphson method, compute $\sqrt{13}$ correct to four decimal places.

3. (a) Solve the following system of equations AX= b where

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

by the Gauss elimination method with partial pivoting.

(b) Approximate the solution of AX = b, where

$$A=\begin{bmatrix}1 & 2 & 4\\ 2 & 1 & 2\\ 4 & 2 & 1\end{bmatrix},\quad b=\begin{bmatrix}-1\\ 5\\ 3\end{bmatrix}$$

with $X^0 = [0,0,0]^{(T)}$ using Gauss-Jacobi iteration method by performing three iterations.

(c) Consider the following table :

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	–1.5	–1.27	–0.98	–0.63	–0.22	0.25

Use Gregory-Newton backward difference interpolation to construct the interpolating polynomial. Estimate f(0.45).

4. (a) Perform three iterations to solve the linear system

$$2x - y = 7$$

$$-x + 2y - z = 1$$

$$-y + 2z = 1$$

using Gauss-Seidel iteration method by taking the initial approximation as (x,y,z) = (0,0,0).

(b) Construct a quadratic polynomial that passes through the points (–2,4), (0,2) and (2,8) by Lagrange interpolation formula.

(c) Show that

(i) $\mu=\left[1+\dfrac{\delta^2}{4}\right]^{1/2}$

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper	: **5866**
Unique Paper Code	: **2352203601**
Name of the Paper	: **DSC-Probability and Statistics**
Name of the Course	: **Bachelor of Arts/Bachelor of Science (Programme) with Mathematics as non-major/ minor**
Semester	: **VI**

~~Duration : 3~~ **Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

All questions carry equal marks.

Use of non-programmable scientific calculator and statistical tables is permitted.

1. (*a*) Consider the following data :

Class	**Frequency**
[1 3)	1
[3 5)	1

P.T.O.

[5 7)	11
[7 9)	21
[9 11)	25
[11 13)	17
[13 15)	9
[15 17)	4
[17 19)	1

Construct an appropriate histogram and comment on any interesting features.

(*b*) Do running times of American movies differ somehow from running times of French movies ? The author investigated this question by randomly selecting 25 recent movies of each type, resulting in the following running times :

94, 90, 95, 93, 128, 95, 125, 91, 104, 116, 162, 102, 90, 110, 92, 113, 116, 90, 97, 103, 95, 120, 109, 91, 138.

Construct a stem-and-leaf plot.

(*c*) Calculate the sample mean, variance and standard deviation from the following data :

203, 130, 160, 180, 195, 132, 145, 211, 105, 145, 153, 152, 138, 87, 99, 93, 119, 129.

2. (*a*) Find the sample space for rolling of two dice and tossing of three coins.

(*b*) Suppose that 55% of all adults regularly consume coffee, 45% regularly consume carbonated soda, and 70% regularly consume at least one of these two products :

(*i*) What is the probability that a randomly selected adult regularly consumes both coffee and soda ?

(*ii*) What is the probability that a randomly selected adult doesn't regularly consume at least one of these two products ?

(*c*) Consider the following probability :

$P(A) = 0.6$, $P(B) = 0.4$, $P(A \cap B) = 0.3$, $P(C) = 0.2$, $P(A \cap C) = 0.15$, $P(B \cap C) = 0.1$, and $P(A \cap B \cap C) = 0.08$.

Find the value of $P(A|B)$, $A(B|A)$, $P(A|B \cup C)$.

3. (*a*) Define Poisson distribution. Find the mean and variance of a Poisson distributed random variable X. Are they equal ?

(*b*) Let the random variable be defined as :

$X = 1$ if a randomly selected vehicle passes an emission test and $X = 0$ otherwise. Assume that the probability mass function (pmf) of this variable is : $p(1) = p$ and $p(0) = 1 - p$.

(*i*) Name the random variable.

(*ii*) Compute $E(X^2)$.

(*iii*) Show that $V(X) = p(1 - p)$.

(*c*) Starting at a fixed time, the gender of each newborn child is observed at a certain hospital until a boy (B) is born. Let p = P (boy is born) = P(B) and assume that the successive births are independent. Let the random variable X = number of births up to and including that of the first boy :

(*i*) Find the probability mass function (pmf) of X.

(*ii*) Determine the cumulative distribution function (cdf) of X.

4. (*a*) Compute the following binomial probabilities directly from the formula for $b(x : n, p)$:

(*i*) $b(3 : 6, 0.5)$

(*ii*) $P(3 \leq X)$ when $n = 6$ and $p = 0.5$.

(*iii*) $P(X \leq 1)$ when $n = 6$ and $p = 0.5$.

(*b*) Find k such that the function f defined by :

$$f(x) = \begin{cases} kx^2, & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function (pdf). Also, determine $P(0 < x < 4)$.

(*c*) Define exponential distribution. Find the mean and standard deviation of an exponential distributed random variable X. Are they equal ?

5. (*a*) "Time headway" in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is essentially the one suggested in "The Statistical Properties of Freeway Traffic". (Transp. Res., vol. 11 : 221–228) :

$$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)}, & x \geq 0.5 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that headway time is at most 5 sec.

(*b*) Let X, the thickness of a certain metal sheet, have a uniform distribution on [A, B]. Write the probability density function (pdf) of X and find the cumulative distribution function (cdf) of X.

(*c*) The amount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size *c* will ensure that overflow occurs only 0.5% of the time ?

6. (*a*) The distribution of egg weights (g) of a certain type is normal with mean value 53 and standard deviation 0.3. Let $X_1, X_2, \ldots\ldots, X_{12}$ denote the weights of a dozen randomly selected eggs; these X_i's constitute a random sample of size 12 from the specified normal distribution. Find the mean and standard deviation of the sample mean weight $\overline{X}$. Also find the probability that the sample mean weight exceeds 53.5 g.

(*b*) Medical researchers have noted that adolescent females are much more likely to deliver low-birth-weight babies than are adult females. Because such babies have higher mortality rates, numerous investigations have focused on the relationship between mother's age and birth weight. For the following data on x = maternal age (yr) and y = baby's birth weight (kg) :

x	y
15	2.3
17	3.4
18	3.3
15	2.6
16	2.9
19	3.3
17	2.9
16	2.5
18	3.1
19	3.6

Compute the coefficient of correlation between x and y. And, how would you describe the nature of relationship between the two variables.

(c) Consider the following data on fracture strength (x, as a percentage of ultimate tensile strength) and attenuation (y, in neper/cm, the decrease in amplitude of the stress wave) in fiberglass-reinforced polyester composites :

x	y
12	3.3
30	3.2
36	3.4
40	3.0
45	2.8
57	2.9
62	2.7
67	2.6
71	2.5
78	2.6

Find the equation of regression line. And estimate the attenuation (y) when fracture strength (x) is 50.

(i) What can be said about the probability that this week's production will be at least 1000?

(ii) What can we say about the probability for week's production between 400 and 600?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2679 **J**

Unique Paper Code : 42357602

Name of the Paper : DSE - Probability and Statistics

Name of the Course : **B.Sc. (Prog.) Physical Sciences / Mathematical Sciences (CBCS-LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all the **six** questions.
3. Attempt any **two** parts from each question.
4. Each part in Question **1, 3, 5** carries **6** marks.
5. Each part in Question **2, 4, 6** carries**6.5** marks.
6. Scientific calculator and standard normal distribution table are allowed.

P.T.O.

1. (a) (i) If the sample space is $C = C_1 \cup C_2$ and if $P(C_1) = 0.8$ and $P(C_2) = 0.5$, find $P(C1 \cap C2)$.

(ii) If C_1 and C_2 are subsets of the sample space C, show that

$P(C_1 \cap C_2) \le P(C_1) \le P(C_1 \cup C_2) \le P(C_1) + P(C_2)$.

(b) Suppose the random variable X has the cumulative distribution function (cdf)

$$F_X(x) = \begin{cases} 0, & if x < -1 \\ \frac{x+2}{4}, & if -1 \le x < 1 \\ 1, & if\ x \ge 1 \end{cases}$$

Sketch the graph of Fx(x). Use your graph to obtain the probabilities:

(i) $P(-1/2 < X \le 1/2)$

(ii) $P(X = 0)$.

(c) If X is a binomial Distribution with parameters n and θ then show that moment generation function of the random variable $\frac{X-\sqrt{n\theta(1-\theta)}}{n\theta}$ approaches that of Standard Normal Distribution when n tends to infinity.

6. (a) If X is a random variable with mean μ and standard deviation σ, then prove that for any $k > 0$,

$$P[|X-\mu| < k\sigma] \ge 1-\frac{1}{k^2}.$$

(b) Let X_i, i = 1 to 5 be independent random variables each being uniformly distributed over (0,1). Use Markov's inequality to get bound on

$$P[X_1 + X_2 + X_3 + X_4 + X_5 \ge 8]$$

(c) Suppose the number of items produced in a factory during a week is a random variable with mean 500 and variance 100.

(b) If the joint probability mass function of X and Y is given by

$$p(x,y)=\begin{cases}\frac{x+2y}{18}, & x=1,2;\quad y=1,2\\ 0 & \text{elsewhere}\end{cases}$$

determine the conditional mean and variance of Y given X = 1.

(c) If X and Y are independent random variables, then E(XY) = E(X)E(Y) and σ_{XY} = 0.

5. (a) Show that for two independent random variables X and Y, E[XY] = E[X], E[Y] and hence, show that Cov(X,Y) = 0. Is the Converse also true?

(b) Given joint pdf for two random variables X and Y

$$f(x,y)=\begin{cases}x.e^{-x(1+y)} & \text{for } x>0 \text{ and } y>0\\ 0 & \text{elsewhere}\end{cases}$$

Find the regression equation of Y on X.

(c) Let the pmf p(x) be positive at x = –1, 0, 1 and zero elsewhere.

(i) If p(0) = 1/4 , find $E(X^2)$.

(ii) If p(0) =1/4 and if E(X) = 1/4, determine p(–1) and p(1).

2. (a) Let a random variable X have the pdf f(x) = 6x(1 – x), 0 < x < 1, zero elsewhere. Compute P(μ – 2σ < X < μ + 2σ), where μ and σ are mean and standard deviation of random variable X respectively.

(b) Define Poisson Distribution. Records show that the probability is 0.00005 that a car will have a flat tire while crossing a certain bridge. Use the Poisson distribution to approximate the binomial probabilities that, among 10,000 cars crossing this bridge,

(i) exactly two will have a flat tire;

(ii) at most two will have a flat tire.

(c) Show that the Normal Distribution has

(i) relative maximum at $x = \mu$ and

(ii) inflection points at $x = \mu - \sigma$ and $x = \mu + \sigma$.

3. (a) Find the joint probability density function (pdf) of the two random variables X and Y whose joint distribution function given by

$$F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}), & x>0,\ y>0\\ 0 & \text{elsewhere}\end{cases}$$

use the joint pdf to find $P(1<X<3,\ 1<Y<2)$ and $P(X + Y \le 1)$.

(b) Let X and Y be two random variables with joint probability mass function $p(x, y) = c(x^2 + y^2)$, $x = -1, 0, 1, 3;\ y = -1, 2, 3$

Find

(i) the value of c

(ii) $P(X \le 1,\ Y > 2)$

(iii) $P(X = 0,\ Y \le 2)$

(c) Let X and Y have the joint probability density function (pdf)

$$f(x,y) = \begin{cases}15x^2y, & 0<x<y<1\\ 0 & \textit{elsewhere.}\end{cases}$$

Then

(i) find marginal densities of X and Y

(ii) compute $P(X + Y \le 1)$.

4. (a) Let X and Y be two random variables with joint probability density function

$$f(x,y)=\begin{cases}xe^{-y}, & 0<x<y<\infty\\ 0 & \text{elsewhere}\end{cases}$$

Determine the joint moment generating function. Does

$$M(t_1, t_2) = M(t_1, 0)M(0, t_2)?$$

This question paper contains 3 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper	:	**8084**
Unique Paper Code	:	**2353010015**
Name of the Paper	:	**Research Methodology**
Name of the Course	:	**B.A. Programme**
Semester	:	**VI**

Duration : **3 Hours** Maximum Marks : **90**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions carry equal marks.

1. (*a*) Describe the format of a mathematical research article/paper. 7.5

   (*b*) What is the role of Mathematics Subject Classification and Keywords in a research paper ? 7.5

   (*c*) In context of mathematical writing, give brief description of a Lemma, Theorem and Conjecture with example. 7.5

2. (*a*) Describe the refereeing process after submission of a manuscript. 7.5

P.T.O.

(*b*) What are the points to be kept in mind while selecting a research topic ? 7.5

(*c*) What are the common errors that may appear in mathematical writing and how to correct them ? 7.5

3. (*a*) Write a LaTeX source program for creating the title page of a research paper. How the sectioning commands help in organizing a research document ? 7.5

(*b*) Discuss the characteristics of good power point presentation. 7.5

(*c*) What is PSTricks ? How is it useful in making simple pictures ? 7.5

4. (*a*) State and discuss the list-making environments in LaTeX. 7.5

(*b*) How to define new command and environment in LaTeX document ? 7.5

(*c*) How to make a beamer presentation ? Explain it with the help of example. 7.5

5. (*a*) List any three features of MAA, AMS and SIAM that support mathematical research. 7.5

(*b*) What is the difference between arXiv and ResearchGate in terms of content and use ? 7.5

(*c*) Identify which web resource you would use for : Finding math journals. connecting with researchers, Reading free preprints (Justify your choices briefly.) 7.5

6. (*a*) A journal published 100 papers in 2021 and 120 in 2022. These received 660 citations in 2023. Calculate its Impact Factor as per JCR and Cite score. 7.5

(*b*) Explain the meaning of SNIP and SJR. Why are they needed alongside the Impact Factor ? 7.5

(*c*) What is plagiarism ? Explain with *two* examples. What software can help detect plagiarism ? 7.5

UNIVERSITY OF DELHI
DELHI-110007

CORRIGENDUM

Sl. No. of Q.P	:	8084
Unique Paper Code	:	2353010015
Name of the Paper	:	Research Methodology
Course	:	B.Sc. (Hons/Prog)
Exam	:	02/06/2025 Morning
SEMESTER/YEAR	:	VI
DURATION	:	3 Hour
Maximum Marks	:	90

Announcement for candidates

Course Name be read as **B.Sc. (Hons/Prog)** instead of B.A. Programme

Joint Registrar (Secrecy)