(d) Suppose f is differentiable on $\mathbb{R}$, $1 \le f'(x) \le 2$ for $x \in \mathbb{R}$, and $f(0) = 0$. Prove $x \le f(x) \le 2x$ for all $x \ge 0$. (5)

5. (a) Let f be differentiable function on an open interval (a, b). Then show that f is strictly increasing on (a, b) if $f'(x) > 0$. (5)

(b) If $y = \sin^{-1} x$, prove that (5)

$$(1 - x^2)y_{n+2} - (2n+1)x\, y_{n+1} - n^2 y_n = 0.$$

(c) If $y = \left[x + \sqrt{1+x^2}\right]^m$, find $y_n(0)$. (5)

(d) State Taylor's theorem. Find Taylor series expansion of cos x. (5)

6. (a) Find all values of k and l such that

$$\lim_{x \to 0} \frac{k + \cos lx}{x^2} = -4.$$ (5)

(b) Determine the position and nature of the double points on the curve (5)

$$y^2 = (x-2)^2(x-1).$$

(c) Sketch a graph of the rational function showing the horizontal, vertical and oblique asymptote (if any) of $y = x^2 - \frac{1}{x}$. (5)

(d) Sketch the curve in polar coordinates of $r = \cos 2\theta$. (5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4121 **H**

Unique Paper Code : 2352011202

Name of the Paper : CALCULUS – DSC 5

Name of the Course : **B.Sc. (H) Mathematics UGCF-2022**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **three** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) If $f: A \to \mathbb{R}$ and if c is a cluster point of A then prove that f can have only one limit at c. (5)

(b) Use $\epsilon - \delta$ definition of limit to establish the following limit: (5)

$$\lim_{x \to -1} \frac{x+5}{2x+3} = 4.$$

(c) Determine whether the following limit exists in $\mathbb{R}$

$$\lim_{x\to 0} sgn \sin\left(\frac{1}{x}\right) \qquad (5)$$

(d) Let $A \subseteq \mathbb{R}$, let $f, g, h: A \to \mathbb{R}$, and let $c \in \mathbb{R}$ be cluster point of A. If $f(x) \le g(x) \le h(x)$ for all $x \in A$, $x \neq c$ and if $\lim_{x\to c} f = L = \lim_{x\to c} h$, then show that $\lim_{x\to c} g = L$. (5)

2. (a) State and prove sequential criterion for continuity. (5)

(b) Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is discontinuous at every x in $\mathbb{R}$. (5)

(c) Let f and g be continuous real-valued function on (a, b) such that $f(r) = g(r)$ for each rational number r in (a, b) then prove that $f(x) = g(x)$ for all $x \in (a, b)$. (5)

(d) State Intermediate Value Theorem. Show that $x = \cos x$ for some x in $\left(0, \frac{\pi}{2}\right)$. (5)

3. (a) Prove that every continuous function defined on a closed interval is bounded therein. (5)

(b) If f is uniformly continuous on a set S and (s_n) is a Cauchy sequence in S, then prove that $(f(s_n))$ is a Cauchy sequence. (5)

(c) Show that the function $f(x) = \frac{1}{x^2}$ is not uniformly continuous on (0,1) but it is uniformly continuous on $[a, \infty)$ where $a > 0$. (5)

(d) Let $f(x) = x^2 \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$. Show that f is differentiable on $\mathbb{R}$ and also show that f′ is not continuous at $x \neq 0$. (5)

4. (a) Let f be defined on an open interval containing x_0. If f assumes its maximum or minimum at x_0 and if f is differentiable at x_0 then show that $f'(x_0) = 0$. (5)

(b) State Intermediate Value Theorem for derivatives. Suppose f is differentiable on E and $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that $f'(x) = \frac{1}{3}$ for some $x \in (0,2)$.

(ii) Show that $f'(x) = \frac{2}{5}$ for some $x \in (0,2)$. (5)

(c) Show that $ex \le e^x$ for all $x \in \mathbb{R}$. (5)

(c) Let W be the subspace of $\mathbb{R}^3$, whose vectors lie in the plane $2x - 2y + z = 0$ and $v = [1, -4, 3] \in \mathbb{R}^3$. Find $\text{proj}_W v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^{\perp}$. (6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1047 **H**

Unique Paper Code : 32355202

Name of the Paper : Linear Algebra

Name of the Course : **Generic Elective-Mathematics [other than Maths (H)]**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

1. (a) If x and y are non-zero vectors in R^n, then prove that (6½)

(i) If $x, y \geq 0$, then $||x + y|| > ||y||$.

(ii) $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$.

(b) If x and y are non-zero vectors in $\mathbb{R}^n$, then x, y = $||x|| \, ||y||$ if and only if y is a positive scalar multiple of x. (6½)

(c) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 6 \\ 3 & 8 & 9 & 10 \\ 2 & -1 & 2 & -2 \end{bmatrix}$ to reduced row echelon form by using elementary row operations. (6½)

(b) Find the kernel and range of the linear transformation $L : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $L([a_1, a_2, a_3, a_4]) = [0, a_2, 0, a_4]$. (6½)

(c) Check whether the linear transformation $L: \mathbb{R}^2 \to \mathbb{R}^2$ given by

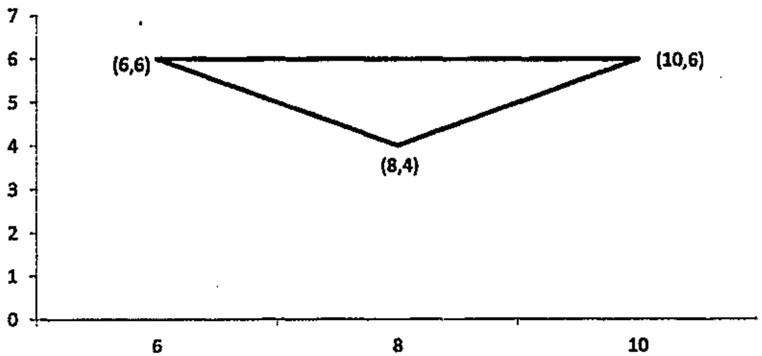
$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -4 & -3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

is one to one or onto. (6½)

6. (a) Find a least square solution for the following inconsistent system (6)

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}$$

(b) Let $W = \text{span}\{[1, -2, -1], [3, -1, 0]\}$. Find a basis for W and its orthogonal complement $W^\perp$. (6)



(i) Translation along the vector (–3,5).

(ii) Scaling about the origin with scale factors of $\frac{1}{2}$ in the x-direction and 3 in the y-direction. (6½)

2. (a) Solve the following system of linear equations using Gauss-Jordan method

$$2x_1 + 3x_2 - x_3 = 9$$

$$x_1 + x_2 + x_3 = 9$$

$$3x_1 - x_2 - x_3 = -1$$

(6)

(b) Show that the matrix $A = \begin{bmatrix} -3 & -1 & -2 \\ -2 & 16 & -18 \\ 2 & 9 & -7 \end{bmatrix}$ is not diagonalizable. (6)

(c) Prove that the set $S = \{[1, 3, -1], [2, 7, -3], [4, 8, -7]\}$ spans $\mathbb{R}^3$. (6)

3. (a) Use the Simplified Span Method to find a simplified general form for all the vectors in span(S) where $S = \{[1, -1, 1], [2, -3, 3], [0, 1, -1]\}$ is a subset of $\mathbb{R}^3$. Is the set S linearly independent? Justify. (6½)

(b) Examine whether the set $S = \{[1, 3, -1], [2, 7, -3], [4, 8, -7]\}$ forms a basis for $\mathbb{R}^3$? (6½)

(c) Define rank of a matrix. Let $A = \begin{bmatrix} -2 & 1 & 8 \\ 7 & -2 & -22 \\ 3 & -1 & -10 \end{bmatrix}$.

Using rank of A determine whether the homogeneous system $AX = 0$ has a non-trivial solution or not. If so, find the non-trivial solution. (6½)

4. (a) Let $S = \{[1, -1], [2, 1]\}$ and $T = \{[3,0], [4, -1]\}$ be two ordered bases for $\mathbb{R}^2$. Find the transition matrix $P_{S \leftarrow T}$ from T-basis to S-basis. Hence, or otherwise find transition matrix from S-basis to T-basis. (6)

(b) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator and $L([1, 0, 0]) = [1, 1, 0]$, $L([0, 1, 0]) = [2, 0, 1]$ and $L([0, 0, 1]) = [1, 0, 1]$. Find $L([x,y,z])$ for any $[x,y,z] \in \mathbb{R}^3$. Also find $L([1, 2, 3])$. (6)

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by $L([x, y, z)] = [-2x + 3z, x + 2y - z]$. Let $B = \{[1, -3, 2], [-4, 13, -3], [2, -3, 20]\}$ and $C = \{[-2, -1], [5, 3]\}$ be a basis for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Find the matrix A_{BC} for L with respect to the given basis B and C. (6)

5. (a) For the graphic in the following figure, use ordinary coordinates in $\mathbb{R}^2$ to find new vertices after performing each indicated operation. Then sketch the figures that would result from each movement.

3

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3489 **H**

Unique Paper Code : 42351201

Name of the Paper : Calculus and Geometry, CBCS (LOCF)

Name of the Course : **B.Sc. (Programme) Mathematical Sciences / Physical Sciences**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. All questions are compulsory.
5. Marks are indicated.

1. (a) Use the second derivative test to determine whether each critical number of the function $f(x) = 3x^5 - 5x^3 + 2$ corresponds to a relative maximum, a relative minimum, or neither. (6)

P.T.O.

(b) Find all horizontal and vertical asymptotes of the graph of the function $f(x)=\frac{1}{x+1}+\frac{1}{x-1}$. (6)

(c) (i) Determine whether the graph of the function $f(x) = x^{1/3}(x-4)$ has a vertical tangent or a cusp. (3)

(ii) Evaluate $\lim\limits_{x\to 1^-}\frac{x-1}{|x^2-1|}$. (3)

2. (a) Sketch the graph of the function $y = x^3 - 3x + 2$. (6.5)

(b) Evaluate the following limits using I'Hopital's rule. (6.5)

(i) $\lim\limits_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$ (3)

(ii) $\lim\limits_{x\to\infty}\left[\cos\left(\frac{2}{x}\right)\right]^{x^2}$ (3.5)

(c) Trace the curve

$x = a(t-\sin t),\ y = a(1+\cos t),\ 0\le t\le 2\pi.$ (6.5)

3. (a) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, show that

$$I_{n-1}+I_{n+1}=\frac{1}{n}$$ (6)

(b) Derive the reduction formula for $\int_0^{\pi/2}\sin^m x\cos^n x\,dx$ and use it to find the value of

$$\int_0^{2a}x^2\sqrt{2ax-x^2}\,dx.$$ (6)

(c) Sketch the graph of $r = a(1 + \cos\theta)$ in polar coordinates, assuming 'a' to be a positive constant. (6)

4. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y=\sqrt{x}$, $x = 1$ and $x = 4$ and the x axis is revolved about the y-axis. (6.5)

(b) Find the arc length of the parametric curve defined by

$$x=\frac{t^3}{3},\ y=\frac{t^2}{2},\ 0\le t\le 1.$$ (6.5)

(c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y axis. (6.5)

5. (a) Sketch the parabola and label the focus, vertex and directrix

$$y^2 - 6y - 2x + 1 = 0.$$ (6)

(b) Describe the graph of the equation.

$$16x^2 - y^2 - 32x - 6y = 57. \qquad (6)$$

(c) Rotate the coordinate axes to remove the xy term. Identify the type of conic and sketch its graph

$$x^2 + 4xy - 2y^2 - 6 = 0. \qquad (6)$$

6. (a) Sketch the graph of the elliptic cone

$$z^2 = x^2 + \frac{y^2}{4}. \qquad (6.5)$$

(b) Define an inverse-square field. Show that the divergence of the inverse-square field

$$F(x,y,z) = \frac{c}{\left(x^2+y^2+z^2\right)^{3/2}}(xi + yj + zk) \text{ is zero.}$$

(6.5)

(c) Find parametric equations of the tangent line to the circular helix

$$x = \cos t, \quad y = \sin t, \quad z = t$$

at $t = t_0 \in \mathbb{R}$, and use that result to find parametric equations for the tangent line at the point $t = \pi$. (6.5)

10

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4020 **H**

Unique Paper Code : 2352571201

Name of the Paper : Elementary Linear Algebra

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of simple calculator is allowed.

P.T.O.

1 (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $||x + y|| \geq |\,||x|| - ||y||\,|$. Also verify it for the vectors $x = [2, -1, 3, 2]$ and $y = [4, 3, 2\ 1]$ in $\mathbb{R}^4$. (5.5+2)

(b) Solve the following system of linear equations using Gaussian Elimination method. Also indicate whether the system is consistent or inconsistent.

$$3x - 2y + 4z = -54$$

$$-x + y - 2z = 20$$

$$5x - 4y + 8z = -83$$

(6.5+1)

(c) Find the quadratic equation $y = ax^2 + bx + c$ that goes through the points (3,18), (2,9) and (-2,13). (7.5)

2. (a) Solve the following system of equations using the Gauss-Jordan method :

$$5x + 20y - 18z = -11$$

$$3x + 12y - 14z = 3$$

$$-4x - 16y + 13z = 13$$

(7.5)

(b) Find the rank of the following matrix by converting to row echelon form :

$$\begin{bmatrix} 8 & 0 & 0 & 1 \\ 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 8 & 1 & 8 \end{bmatrix}$$

(7.5)

(c) Find the characteristic polynomial, eigenvalues and corresponding eigenvectors for the given matrix :

$$\begin{bmatrix} -1 & 2 \\ 4 & -3 \end{bmatrix}$$

(2+2+3.5)

3. (a) Use the Diagonalization Method to determine whether the following matrix is diagonalizable.

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

(7.5)

(b) Show that the set M_{22} of all 2×2 matrices is a vector space under the usual operations of addition and scalar multiplication. (7.5)

(c) Show that span (S) is a subspace of V, where S a nonempty subset of a vector space V. Let P_3 be the vector space of all real polynomials of degree ≤ 3 and $S = \{x, x^2 + 1, x^3 - 1\}$. Find span (S). Does $(x^2 + x^3) \in$ span (S)? (3+3+1.5)

4. (a) Check whether the following subset of R is linearly independent or not.

$$S = \{(0,1,-1), (1,1,0), (1,0,2)\}.$$

Express (1,1,1) as linear combination of vectors in S. (4+3.5)

(b) Define a basis of a vector space. Show that the following set S is a basis for the vector space M_{22} of all 2×2 matrices :

$$S = \left\{ \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 5 & -2 \\ 0 & -3 \end{pmatrix} \right\}$$ (2+5.5)

(c) Define a finite dimensional vector space. Let W be the solution set to the matrix equation $AX = 0$, where $A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Show the following :

(i) W is a subspace of R^3.

(ii) Find a basis for W. (1.5+3+3)

5. (a) Check if the following mappings are linear transformation or not. Prove it or give a counter example to disprove.

(i) f: $R^3 \to R^3$ defined as $f([a, b, c]) = [-b, c, 0]$

(ii) g: $M_{22} \to R$ defined as $g\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$,

where M_{22} is the vector space of 2×2 real matrices. (4+3.5)

(b) Find the matrix T_{AB} of linear transformation T: $R^2 \to R^3$ with respect to basis B = {(1,0), (0,1)} and C = {(1,1,0), (0,1,1), (1,0,1)}, where

$$T([a,b]) = [a - b, a, 2a + b] \quad (7.5)$$

(c) Define kernel of a linear transformation T. Show that Ker(T) = {0} if and only if T is one-one. (2+5.5)

6. (a) Consider the linear operator L : $M_{33} \to M_{33}$, defined as $L(A) = A - A^T$, where M_{33} is vector space of 3×3 matrices and A^T denotes the transpose of the matrix A.

Find dim(Ker(L)) + dim(Range(L)). (7.5)

(b) Define an onto linear transformation. If T be a linear transformation from a finite dimensional vector space V to a finite dimensional vector space W, show that T is onto if and only if dim (Range(L)) = dim (W). (2+5.5)

(c) Show that the mapping f: $M_{nm} \rightarrow M_{mn}$ defined as $f(A) = A^T$ is an isomorphism. (7.5)

3

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4083 **H**

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) If x and y are vectors in $\mathbb{R}^n$, then prove that $|x \cdot y| \leq \|x\|\,\|y\|$. Also, verify the same for the vectors $x = [4,2,0,-3,-1]$ and $y = [1,4,-1,0,2]$ in $\mathbb{R}^5$.

P.T.O.

(b) Using the Gauss – Jordan method, find the complete solution set for the following non-homogeneous system of linear equations :

$$2x_1 + 4x_2 - x_3 = 9$$

$$3x_1 - x_2 + 5x_3 = 5$$

$$8x_1 + 2x_2 + 9x_3 = 19$$

(c) Define the row space of a matrix. Also, determine whether the vector v = [7,1,18] is in the row space of the following matrix :

$$A = \begin{pmatrix} 3 & 6 & 2 \\ 2 & 10 & -4 \\ 2 & -1 & 4 \end{pmatrix}$$

If so, then express [7,1,18] as a linear combination of the rows of A.

2. (a) Define the rank of a matrix. Consider the matrix :

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 5 & -4 & 3 \end{pmatrix}$$

Using rank of A, determine whether the homogeneous system AX = 0 has a non-trivial solution or not.

(b) Consider the matrix :

$$A = \begin{pmatrix} 4 & 0 & 4 \\ 0 & 4 & 4 \\ 4 & 4 & 8 \end{pmatrix}.$$

(i) Find the eigenvalues and the fundamental eigenvectors of A.

(ii) Is A diagonalizable? Justify your answer.

(c) Find the quadratic equation of the form $y = ax^2 + bx + c$ that passes through the points $(-2,20)$, $(1,5)$, and $(3,25)$.

3. (a) (i) Let $\mathcal{C}$ be a family of subspaces of a vector space V and let W denote the intersection of subspaces in $\mathcal{C}$. Prove that W is a subspace of V.

(ii) Let F be any field. Prove that $W = \{(a_1, a_2, \ldots, a_n) \in F^n \mid a_1 + a_2 + \cdots a_n = 0\}$ is a subspace of F^n.

(b) Let W_1 and W_2 be subspaces of a vector space V. Prove that V is the direct sum of W_1 and W_2 if and only if each vector in V can be uniquely written as $x_1 + x_2$, where $x_1 \in W_1$ and $x_2 \in W_2$.

(c) (i) Show that if S_1 and S_2 are arbitrary subsets of a vector space V_1 then

$$\text{span}(S_1 \cap S_2) \subseteq \text{span}(S_1) \cap \text{span}(S_2).$$

Give an example in which $\text{span}(S_1 \cap S_2)$ and $\text{span}(S_1) \cap \text{span}(S_2)$ are unequal.

(ii) Does the polynomial $-x^3 + 2x^2 + 3x + 5$ belong to span (S), where

$$S = \{x^3 + x + 1,\ x^3 - 2x^2 + 1,\ x^2 + x + 1).$$

4. (a) Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Prove that $S \cup \{v\}$ is linearly dependent if and only if $v \in \text{span}(S)$.

(b) Let $V = M_{2\times2}(F)$, $W_1 = \left\{\begin{pmatrix} a & b \\ c & a \end{pmatrix} \in V \,\middle|\, a,b,c \in F\right\}$ and

$$W_2 = \left\{\begin{pmatrix} 0 & a \\ -a & b \end{pmatrix} \in V \,\middle|\, a,b \in F\right\}.$$

Find a basis for the subspaces W_1, W_2 and $W_1 \cap W_2$. Also, find the dimension of each of them.

(c) Let W be a subspace of a finite dimensional vector space V. Prove that W is finite dimensional and $\dim(W) \leq \dim(V)$. Further, if $\dim(W) = \dim(V)$, then show that $V = W$.

5. (a) Let V and W be two finite dimensional vector spaces and let $T: V \to W$ be a linear transformation. Then prove that

$$\text{nullity}(T) + \text{rank}(T) = \dim(V)$$

Also, prove that if $\dim(V) < \dim(W)$, then T cannot be onto.

(b) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(a_1, a_2, a_3) = (-2a_1 + 3a_3, a_1 + 2a_2 - a_3)$ with $\beta = \{(1,-3,2), (-4,13,-3), (2,-3,20)\}$ and $\gamma = \{(-2,-1), (5,3)\}$. Compute $[T]^{\gamma}_{\beta}$. Is T one to one?

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by :

$$L\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 + u_3 \\ u_1 + u_2 \\ u_2 - u_3 \end{pmatrix}$$

Find bases for null space N(T) and range space R(T). Also, verify the dimension theorem.

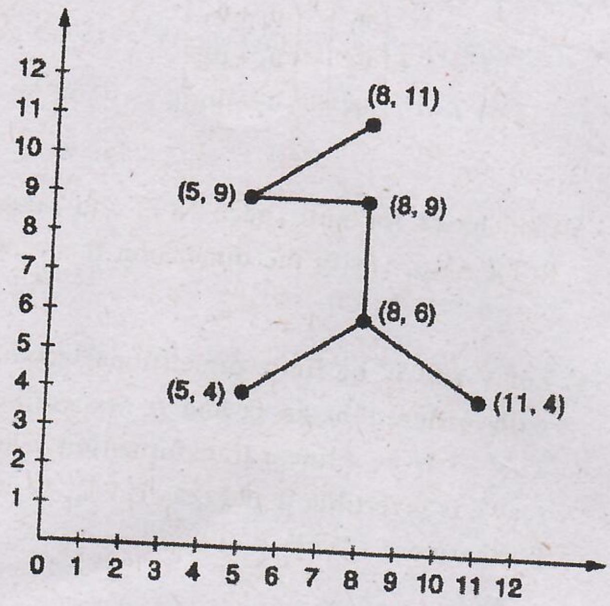
6. (a) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let $T: V \to W$ be a linear transformation. Then prove that T is invertible if and only if $[T]^{\gamma}_{\beta}$ is invertible. Furthermore, $[T^{-1}]^{\beta}_{\gamma} = ([T]^{\gamma}_{\beta})^{-1}$.

(b) Let V and W be finite dimensional vector spaces and let $T: V \to W$ be an isomorphism. Let V_0 be a subspace of V.

(i) Prove that $T(V_0)$ is a subspace of W.

(ii) Prove that $\dim(V_0) = \dim(T(V_0))$.

(c) For the adjoining graphic, use homogenous co-ordinates to find the new vertices after performing scaling about (8,4) with scale factors of 2 in the x-direction and $^1/_3$ the y-direction.



Also, sketch the final figure that would result from this movement.

(c) A large tank contains 100 litres of salt water. Initially s_0 kg of salt is dissolved. Salt water flows into the tank at the rate of 10 litres per minute, and the concentration $c_{in}(t)$ (kg of salt/litre) of this incoming water-salt mixture varies with time. We assume that the solution in the tank is thoroughly mixed and that the salt solution flows out at the same rate at which it flows in: that is, the volume of water-salt mixture in the tank remains constant. Find a differential equation for the amount of salt in the tank at any time t.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4159 **H**

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. Each part carries **7.5** marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve

$$\{y^2(x+1)+y\}dx + (2xy+1)dy = 0$$

(b) Solve the initial value problem

$$x\frac{dy}{dx}\,y = y^2 \log x,\ \ y(1) = 2$$

(c) (i) Solve

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

(ii) Solve by reducing the order $y'' = (x + y')^2$

2. (a) Suppose that a mineral body formed in an ancient cataclysm originally contained the uranium isotope ^{238}U, (which has a half-life of 4.51×10^9 years) but no lead, the end product of the radioactive decay of ^{238}U. If today the ratio of ^{238}U atoms to lead atoms in the mineral body is 0.9, when did the cataclysm occur?

(b) Upon the birth of their first child, a couple deposited Rs. 10,000 in an account that pays 8% interest compounded continuously. The

6. (a) The Earth's atmospheric pressure p is often modelled by assuming that $\frac{dp}{dx}$ (the rate at which pressure p changes with altitude h above sea level) is proportional to p. Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 20 km is 50 millibars. Use an exponential decay model

$$\frac{dp}{dx} = kp$$

to describe the system, and then by solving the equation, find an expression for p in terms of h. Determine k and the constant of integration from the initial conditions. What is the atmospheric pressure at an altitude of 50 km?

(b) Discuss Phase Plane Analysis of predator-prey model.

predators that compete for a single prey food-source is

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ, \frac{dY}{dt} = a_2XY - b_2Y, \frac{dZ}{dt} = a_3XZ - b_3Z$$

where a_i, b_i, c_i for i = 1,2,3 are all positive constants. Here X(t) is the prey density and Y(t), Z(t) are the two predator densities. Find all possible equilibrium populations.

(c) Suppose a population can be modeled using the differential equation

$$\frac{dX}{dt} = 0.2X - 0.001X^2$$

with an initial population size of $x_0 = 100$ and a time step of 1 month. Find the predicted population after 2 months.

interest payment is allowed to accumulate. In how many years will the amount double? How much will the account contain on the child's 18th birthday?

(c) A roast initially at 50°F, is placed in a 375°F oven at 5 pm. After 75 minutes, it is found that the temperature of the roast is 125°F. When will the roast be 150°F?

3. (a) Show that the solutions e^x, e^{-x}, e^{-2x} of the third order differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

are linearly independent. Find the particular solution satisfying the given initial condition.

$$y(0) = 1,\ y'(0) = 2,\ y''(0) = 0$$

(b) Solve the differential equation using the method of Variation of Parameters

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=2e^{2x}$$

(c) Find the general solution of the differential equation using the method of Undetermined Coefficients.

$$\frac{d^2y}{dx^2}+9y=2\cos 3x$$

4. (a) Use the operator method to find the general solution of the following linear system

$$\frac{dx}{dt}+\frac{dy}{dt}-2x-4y=e^{t}$$

$$\frac{dx}{dt}+\frac{dy}{dt}-y=e^{4t}$$

(b) Find the general solution of the differential equation. Assume $x>0$.

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=72x^5$$

(c) A body with mass $m=\frac{1}{5}$ kg is attached to the end of a spring that is stretched 4m by a force of 20N. It is set in motion with initial position $x_0=1$m and initial velocity $v_0=-5$m/s. Find the position function of the body as well as the amplitude, frequency and period of oscillation.

5. (a) Develop a model with two differential equations describing a predator-prey interaction.

(b) Define equilibrium solution of differential equation. A model of a three species interaction with two

(c) For given initial value problem (IVP)

$$\frac{dy}{dx} = y - x, \; y(0) = 2,$$

find y(0.5) and y(1.0) by using the Heun's method. Also, find the absolute error at each step, given that the exact solution of the IVP is $y(x) = e^x + x + 1$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1162 **H**

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : **CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Use of scientific calculator is allowed.

1. (a) Convert $(0.7)_{10}$, given in the decimal number system, to the binary number system with 7 significant digits. (6)

(b) Write the approximate value of the number $\frac{5}{3}$ with four significant digits and evaluate round-off error, relative error and absolute percentage error. (6)

(c) Find the approximate root of equation $x^3 + x - 1 = 0$ correct up to three decimal places by using the secant method on the interval (0,1). (6)

2. (a) Find the approximate root of the equation $x^4 - 3x + 1 = 0$ by using Newton-Raphson's method by taking initial approximation $x_0 = 1.5$. Perform three iterations of the method. (6.5)

(b) Find the approximate root of the equation $x^2 - 2x - 1 = 0$ in the interval (2,3) by using the Regula-Falsi method. Perform three iterations of the method. (6.5)

(c) Use the formula

$$f'(x) \approx \frac{-3f(x_i) + 4f(x_i + h) - f(x_i + 2h)}{2h}$$

to approximate the derivative of $f(x) = \ln x$ at $x_i = 2$, taking $h = 1$, 0.1 and 0.01. (6)

6. (a) Approximate the value of $(\ln 2)^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$ by using the Trapezoidal rule with $h = 0.25$. (6.5)

(b) Apply Euler's method to approximate the solution of the initial value problem :

$$\frac{dy}{dx} = \frac{1+y^2}{x}, \quad 1 \le x \le 4, \quad y(1) = 0,$$

by using 5 steps. (6.5)

$$f'(x_i) \approx \frac{f(x_i) - f(x_i - h)}{h} \text{ and}$$

$$f''(x_i) \approx \frac{f(x_i - 2h) - 2f(x_i - h) + f(x_i)}{h^2}.$$

x	0.60	0.65	0.70	0.75
$f(x)$	0.6221	0.6155	0.6138	0.6170

(b) Given the function $f(x) = 1 + x + x^3$, approximate $f'(1)$ with Richardson extrapolation by using first order forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \text{ with } h = 0.25 \text{ and } 0.125.$$

(6)

(c) Find the approximate root of the equation $\cos x - xe^x = 0$ in the interval $(0,1)$ by using the bisection method. Perform four iterations of the method. (6.5)

3. (a) Using Gauss Jordan's method solve the following system of linear equations : (6)

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16.$$

(b) Show that $1 - E^{-1} = -\frac{1}{2}\delta^2 + \delta\mu$. (Note: Symbols have their own meaning) (6)

(c) Find the Newton interpolating polynomial which fits into the given data. (6)

x	1	2	7	8
f(x)	1	5	5	4

4. (a) By using the initial solution (0,0,0), perform three iterations of the Gauss Jacobi's method for the following system of linear equations : (6.5)

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110.$$

(b) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the given data and by using it estimate the value of f(1.5). (6.5)

x	0	1	2	3
f(x)	1	2	5	10

(c) Following table gives the area of the circle with given diameter : (6.5)

Diameter (in cm)	80	85	90	95	100
Area (in cm square)	5026	5674	6362	7088	7854

Make the difference table. Obtain the backward Gregory-Newton interpolating polynomial and estimate the area of the circle with diameter 82 cm.

5. (a) For the given below data, find f′(0.7) and f″(0.7) by using forward difference formulae (6)

12

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2875 **H**

Unique Paper Code : 32353401

Name of the Paper : SEC 2 – Computer Algebra Systems and Related Softwares

Name of the Course : **CBCS-LOCF – B.Sc. (H) Mathematics**

Semester : IV

Duration : 2 Hours Maximum Marks : 38

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. This question paper has **four** questions in all.

3. **All** questions are compulsory.

4. Use anyone of the CAS := Mathematica/Maple/ Maxima/any other to answer the questions.

P.T.O.

1. Attempt both parts (i) and (ii).

(i) Fill in the blanks : (1×5=5)

(a) The line numbers assigns to the output as ________.

(b) To clear the variables, use the command ________.

(c) The command to calculate the constant e to 100 decimal places is ________.

(d) % is used for ________.

(e) The command to calculate the factor of 346849 is ________.

(ii) Explain any **FIVE** of the following 'R' commands in short : (1×5=5)

(a) qqline()

(b) stack()

(c) objects()

(d) remove()

(e) as.factor()

(f) head()

2. Write a short note on any **four** from the following : (2×4=8)

(i) How to create a two-dimensional display of the data with headings in any CAS.

(ii) How to display two graphs side by side in any of the CAS.

(iii) How to define a function of two or more variables in any CAS. Give an example of the input of such definition.

(iv) How to plot a 3-dimensional parametric curve. Explain with an example.

(v) How to form a new matrix from two existing matrices of same order by stacking them side by side in any CAS.

(vi) In any of the CAS how to create a specified ordered matrix with all the nonzero entries at the specified positions and rest entries are zero.

3. Do any **four** from the following : (2×4=8)

(i) Write command for solving the system of equations :

$$2x + 3y + 4z = 5;$$

$$x + y + z = 2;$$

$$4x + 2y - z = 1$$

(ii) Write command for sketching the curve:

$$y^2 = 4ax \text{ for } 0 \le x \le 5$$

The colour of the curve is red.

(iii) Write the command for plotting the graph of

$$\sin(x^2 + y^2)\, e^{-x^2} + \cos(x^2 + y^2),\ 0 \le x \le 2,\ 0 \le y \le 2.$$

(iv) Write the output of the following commands in the statistical software 'R'


```
>response=c(5,6,9,12,8,7,9,13,10)
>predictor=c(rep('open',5),rep('closed',4))
>res_pre=data.frame(response,predictor)
>res_pre
>res_pre_m=as.matrix(res_pre)
```

(v) $A = \begin{bmatrix} 1 & 5 & 6 \\ 4 & 9 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Write commands for generating the above matrix and finding its transpose.

4. Attempt any **three** parts from the following : (4×3=12)

(i) Consider two data sets :

D1: 13, 14,21, 18, 5, 24, 10

D2: 10, 12, 18, 9, 11,26, 15

Write code of the following in software- R:

(a) Draw scatter plot of datapoints (D1, D2).

(b) Create dataframe of the above data.

(c) Convert the dataframe into matrix

(ii) Write code of the following in software- R :

(a) Create a vector

y: 5, 8, 13, 20, NA, -3, 0, NA, 15,-31

(b) Find the length of the vector y.

(c) Find Mean of y by dropping NA.

(d) Find the quartile of vector y.

(iii) Write code of the following in software- R:

(a) Create the following dataframe 'd'

6	8	4	9	5	3
4	7	2	1	8	9
3	5	6	8	2	1

(b) Generate a five number summary of d.

(c) Find Mean of d.

(d) Create a box plot for d.

(iv) Consider the following matrix 'y'

	C1	C2	C3	C4	C5
R1	24	15	53	28	1
R2	5	7	35	55	9
R3	19	9	1	6	17
R4	10	14	56	3	32
R5	23	2	12	45	5
R6	34	18	9	3	18

Write possible R commands for the following questions :

(a) Change row name "R3" to "Row3" of matrix 'y'.

(b) Extract rows and columns of 'y', find the mean and standard deviation of each row.

(c) Covert matrix 'y' into data frame.

(d) Find mean of the vector "Row 3" of the converted data frame.

Unique Paper Code : 2351401

Name of the Paper : 32351401

Type of the Paper : Partial Differential Equations (CBCS-LOCF)

Semester : DSC

Programme : IV

Duration : 3 Hours : B. Sc. (H) Mathematics

Instruction for Candidates

Maximum Marks : 75

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Marks of each part are indicated.

Section – I

Q.1. Attempt any two of the following: [7.5+7.5]

a. Find the partial differential equation arising from the following

$$(x-a)^2+(x-b)^2+z^2=r^2$$

where a and b are arbitrary constants.

b. Find the solution of the following Cauchy problem

$$xu_x+yu_y=xe^{-u}$$

with the data $u=0$ on $y=x^2$.

c. Apply the method of separation of variables $u(x,y)=f(x)g(y)$ solve the following equation

$$y^2u_x^2+x^2u_y^2=(xyu)^2.$$

Section – II

Q.2. Attempt any one out of the following: [6]

a. Derive one-dimensional wave equation of the vibrating string

$$u_{tt}=c^2\,u_{xx}+F$$

where, $c^2=\frac{T}{\rho}$, and T is the tension at the end points of the string.

$F=\frac{f}{\rho}$, and f be the external force, acting on the string.

b. Derive the Laplace equation $\nabla^2 V=0$, where ∇^2 is known as Laplace operator.

Q.3. Attempt any two out of the following: [6+6]

a. Classify the second order partial differential equation

$$u_{xx}+x^2u_{yy}=0$$

and transform into canonical form.

b. Find the general solution of the following equation

$$3u_{xx}+10u_{xy}+3u_{yy}=0.$$

c. Apply a linear transformation $\xi=ax+by$ and $\eta=cx+dy$, to transform the Euler equation

$$eu_{xx}+2fu_{xy}+gu_{yy}=0$$

into canonical form where a,b,c,d,e,f and g are constants.

Section – III

Q.4. Attempt any three out of the following: [7+7+7]

a. Determine the solution of the given initial-value problem.

$$u_{tt} - c^2 u_{xx} = e^x, \qquad u(x,0) = 5,\ u_t(x,0) = x^2.$$

b. Obtain the solution of the initial boundary value problem:

$$\begin{aligned} &u_{tt} = 4u_{xx}, && 0 < x < \infty,\ t > 0, \\ &u(x,0) = x^4, && 0 \le x < \infty, \\ &u_t(x,0) = 0, && 0 \le x < \infty, \\ &u(0,t) = 0, && t \ge 0. \end{aligned}$$

c. Obtain the solution of the given problem

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, \\ &u(x,t) = f(x) \quad on \quad x - ct = 0; \\ &u(x,t) = g(x) \quad on \quad t = t(x), \\ &where\ f(0) = g(0). \end{aligned}$$

d. Obtains the solution of

$$\begin{aligned} &u_{xx} - u_{yy} = 1, \\ &u(x,0) = \sin x, \\ &u_y(x,0) = x. \end{aligned}$$

Section – IV

Q.5. Attempt any three out of the following: [7+7+7]

a. Determine the solution of the initial boundary-value problem

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, \quad 0 < x < 1,\ t > 0, \\ &u(x,0) = x(1-x),\ u_t(x,0) = 0, \quad 0 \le x \le 1, \\ &u(0,t) = u(1,t) = 0,\ t > 0. \end{aligned}$$

b. Prove the uniqueness of the solution of the initial boundary-value problem:

$$\begin{aligned} &u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi,\ t > 0, \\ &u(x,0) = f(x),\ u_t(x,0) = g(x), \quad 0 \le x \le \pi, \\ &u_x(0,t) = 0,\ u_x(\pi,t) = 0,\ t > 0. \end{aligned}$$

c. Solve the problem:

$$\begin{aligned} &u_{tt} = c^2 u_{xx} + Ax, \quad 0 < x < 1,\ t > 0, A = constant, \\ &u(x,0) = 0,\ u_t(x,0) = 0, \quad 0 \le x \le 1, \\ &u(0,t) = 0,\ u(1,t) = 0,\ t > 0. \end{aligned}$$

d. Obtain the solution of the initial boundary-value problem

$$\begin{aligned} &u_t = k\,u_{xx}, \quad 0 < x < l,\ t > 0, \\ &u(x,0) = \sin\left(\frac{\pi x}{2l}\right),\ 0 \le x \le l, \\ &u(0,t) = 0,\ u(l,t) = 1,\ t \ge 0. \end{aligned}$$

14

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3365 **H**

Unique Paper Code : 42354401

Name of the Paper : Real Analysis

Name of the Course : **B.Sc. (Prog) Physical Sciences / Mathematical Sciences**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Let A and B be non- empty bounded subsets of R. Let $A + B = \{a + b: a \in A, b \in B\}$.

   Prove that $\inf (A+B) = \inf A + \inf B$.

   (b) Define denumerable set. Show that the set $N \times N$ is a denumerable set.

P.T.O.

(c) Let S be a bounded set in R and let S_0 be a non-empty subset of S. Show that $\inf S \le \inf S_0 \le \sup S_0 \le \sup S$. (6,6)

2. (a) (i) Define the convergence of a sequence (x_n) of real numbers.

(ii) Let (x_n) be a sequence of real numbers that converges to x. Show that the sequence $(|x_n|)$ of absolute values converges to $|x|$.

(b) Prove that if $c > 0$, then $\lim\limits_{n\to\infty} c^{1/n} = 1$.

(c) Show that if $X = (x_n)$ and $Y = (y_n)$ are sequences of real numbers converging to x and y respectively, then their product $X \cdot Y$ converges to xy. (6.5,6.5)

3. (a) State Monotone Convergence Theorem. Show that the sequence (x_n) defined by

$$x_1 = 1;\ x_{n+1} = \sqrt{2+x_n},\ \forall n \in N \text{ is convergent.}$$

Also, find $\lim\limits_{n\to\infty} x_n$.

(b) Define subsequence of a sequence (x_n) of real numbers. Prove that if every subsequence of $X = (x_n)$ has a subsequence that converges to 0, then $\lim X = 0$.

(c) Prove that a Cauchy sequence of real numbers is bounded. Is the converse true? Justify your answer. (6.5,6.5)

4. (a) State and prove Limit Comparison Test for positive term series. Hence, show that the following series converges :

$$\frac{\sqrt{2}}{3.5}+\frac{\sqrt{4}}{5.7}+\frac{\sqrt{6}}{7.9}+\frac{\sqrt{8}}{9.11}+\cdots$$

(b) Check the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \sin\left(\frac{n\pi}{2}\right)$

(ii) $\sum_{n=1}^{\infty} (-1)^n\left(\frac{n}{2n-1}\right)$

(iii) $\sum_{n=1}^{\infty} \left(n^{\frac{1}{n}}-1\right)^n$

(c) State and prove Cauchy's n^{th} Root test for a series of non-negative real numbers. (6,6)

5. (a) Prove that an absolutely convergent series is convergent. Is the converse true? Justify your answer.

(b) Define a power series. Determine the radius of convergence and exact interval of convergence of the power series $\sum \frac{x^n}{n^n}$.

(c) State Weierstrass M-Test for uniform convergence of series. Hence, show that

$$\sum \frac{1}{n^3 + n^4 x^4}, \quad \forall\, x \in R$$

is uniformly convergent. (6.5,6.5)

6. (a) Show that $f(x) = k \ \forall\, x \in [a, b]$ is Riemann integrable, where k is a constant.

(b) Show that the sequence $\left(\frac{x^2 + nx}{n}\right)$ is pointwise convergent but not uniformly convergent on R.

(c) Find the upper and lower Darboux integrals for $f(x) = x^3$ on the interval $[0, b]$. Is f integrable on $[0, b]$? (6,6)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4014 H

Unique Paper Code : 2352572401

Name of the Paper : Abstract Algebra

Name of the Course : B.A./B.Sc. (Programme) with Mathematics as Non-Major/ Minor – DSC

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Each part carries **7.5** marks.
4. Use of Calculator not allowed.

P.T.O.

1. (a) Let n be a fixed positive integer greater than 1. If a mod n = a' and b mod n = b', then prove that $(a+b)\text{mod } n = (a'+b')\text{mod } n$ and $(ab)\text{mod } n = (a'b')\text{mod } n$.

(b) Define a Group. Show that the set {5,15, 25, 35} is a group under multiplication modulo 40. What is the identity element of this group? Is there any relationship between this group and U(8)?

(c) Show that G = GL(2, $\mathbb{R}$), group of 2×2 matrices with real entries and nonzero determinants is non abelian. Also, construct a Cayley table for U(12).

2. (a) Let G be a group and H a nonempty subset of G. Prove that if ab^{-1} is in H whenever a and b are in H, then His a subgroup of G. Also, find the order of 7 in U(15).

(b) Let G be a group and let $a \in G$. Prove that a and a^{-1} have the same order. Illustrate the above result in the group $\mathbb{Z}_{10}$.

(c) Let a be an element of order n in a group G and let k be a positive integer. Show that

$$\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle \text{ and } |a^k| = \frac{n}{\gcd(n,k)}.$$

3. (a) If $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{bmatrix}$.

Show that $\alpha\beta \neq \beta\alpha$. Also compute the value of and find order of $\beta^{-1}\alpha\beta$.

(b) Construct a complete Cayley table for D_4, the group of symmetries of a square. Find the inverse of each of the element in D_4.

(c) Let H = {(1), (1 2)(3 4), (1 3)(2 4), (1 4)(2 3)}. Find all the left cosets of H in A_4, the alternating group of degree 4.

4. (a) State Lagrange's theorem for finite groups. Show that a group of prime order is cyclic.

(b) State the normal subgroup test. Prove that SL(2, $\mathbb{R}$), the group of 2 × 2 matrices with determinant 1 is a normal subgroup of GL(2, $\mathbb{R}$), the group of 2 × 2 matrices with non zero determinant.

(c) Let f: G → G' be a group homomorphism. Prove that if H be a cyclic subgroup of G then f(H) is a cyclic subgroup of G'. Prove or disprove that the map g: ($\mathbb{R}$, +) → ($\mathbb{R}$, +) given by $g(x) = x^2$ is a homomorphism.

5. (a) (i) Prove that the set $A = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ is a subring of the ring of all 2×2 matrices over $\mathbb{Z}$.

(ii) Prove that the ring $\mathbb{Z}_p$ of integers modulo p where p is a prime, is an integral domain.

(b) Define characteristic of a ring. Let R be a ring with unity 1. Prove that if 1 has infinite order under addition then charR = 0. If 1 has order n under addition, then charR = n.

(c) Prove that intersection of any collection of subrings of a ring R is a subring of R. Does the result hold for the union of subrings? Justify.

6. (a) State the ideal test. Let $\mathbb{R}[x]$ denote the set of all polynomials with real coefficients. Let $A = \{p(x) \in \mathbb{R}[x] | \ p(0) = 0\}$. Prove that A is a principal ideal of $\mathbb{R}[x]$.

(b) Determine all the ring homomorphisms from $\mathbb{Z}_n$ to itself.

(c) State and prove the First Isomorphism Theorem for rings.

17

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 4064 **H**

Unique Paper Code : 2352012401

Name of the Paper : Sequence and Series of Functions

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory and carry equal marks.

1. (a) Define uniform convergence of a sequence of functions (f_n) defined on $A \subseteq \mathbb{R}$ to $\mathbb{R}$. If $A \subseteq \mathbb{R}$ and $\varphi: A \to \mathbb{R}$ then define the uniform norm of

φ on A. Discuss the uniform and pointwise convergence of the sequence (f_n), where

$$f_n(x) = \frac{x}{n} \text{ for } x \in \mathbb{R} \text{ and } n \in \mathbb{N}.$$

(b) Let (f_n) be the sequence of functions defined by

$$f_n(x) = \frac{1}{1+x^n} \quad \forall x \in [0,1],\ n \in \mathbb{N}.$$

Find the pointwise limit of the sequence (f_n). Does (f_n) converges uniformly? Justify your answer.

(c) Show that if (f_n) and (g_n) are two sequences of bounded functions on $A \subseteq \mathbb{R}$ to $\mathbb{R}$ that converge uniformly to f and g respectively then prove that the product sequence $(f_n g_n)$ converge uniformly on A to fg. Give an example to show that in general the product of two uniformly convergent sequence may not be uniformly convergent.

2. (a) Let (f_n) be a sequence of integrable functions on [a, b] and suppose that (f_n) converges uniformly to f on [a, b]. Show that f is integrable on [a, b] and

$$\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$$

(b) Let $f_n(x) = \frac{x^n}{n}$ for $x \in [0, 1]$. Show that the sequence (f_n) of differentiable functions converges uniformly to a differentiable function f on [0, 1] and that the sequence (f_n') converges on [0, 1] to a function g but the convergence is not uniform.

(c) Show that the sequence $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on $[0,2]$.

3. (a) State and prove Weierstrass M-Test for uniform convergence of series of functions.

(b) Show that the series $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ is uniformly convergent on $[-a, a]$, $a > 0$ but is not uniformly convergent on $\mathbb{R}$.

(c) Discuss the pointwise convergence of the series of functions $\sum_{n=1}^{\infty} \frac{x^n}{2+3x^n}$ for $x \geq 0$.

4. (a) Let f_n be continuous function on $D \subseteq \mathbb{R}$ to $\mathbb{R}$ for each $n \in N$ and $\sum_{n=1}^{\infty} f_n$ converges uniformly to f on D. Prove that f is continuous on D.

(b) Show that the series of functions $\sum_{n=1}^{\infty} \frac{\cos(x^2+1)}{n^3}$ converges uniformly on $\mathbb{R}$ to a continuous function.

(c) Given the Riemann integrable functions $f_n : \mathbb{R} \to \mathbb{R}$, $n \in N$, such that

$$f_n(x) = \sin\left(\frac{x}{n^4}\right) \text{ for all } x \in \mathbb{R}$$

Show that $\sum_{n=1}^{\infty} \int_{-2\pi}^{2\pi} f_n(x)\, dx = \int_{-2\pi}^{2\pi} \sum_{n=1}^{\infty} f_n(x)\, dx$.

5. (a) Define the radius of convergence and interval of convergence of power series. Check the uniform convergence of the following power series on $[-1, 1]$

$$x+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\frac{x^4}{4^2}+\ldots\ldots\ldots\ldots.$$

(b) State and Prove Cauchy-Hadamard Theorem.

(c) Suppose that $f(x)=\sum_{n=0}^{\infty}a_n x^n$ has radius of convergence $R>0$. Then prove that the series

$\sum_{n=0}^{\infty}\frac{a_n}{n+1}x^{n+1}$ has also radius of convergence R

and for $|x|<R$

$$\int_0^x f(t)dt = \sum_{n=0}^{\infty}\frac{a_n}{n+1}x^{n+1}.$$

6. (a) For cosine function C(x) and sine function S(x) prove the following :

(i) If $f: \mathbb{R} \to \mathbb{R}$ is such that $f''(x) = -f(x)$ for $x \in \mathbb{R}$ then there exist real numbers α and β such that $f(x) = \alpha C(x) + \beta S(x)$ for $x \in \mathbb{R}$.

(ii) If $x \in \mathbb{R}, x \geq 0$ then

$$1-\frac{1}{2}x^2 \leq C(x) \leq 1-\frac{1}{2}x^2+\frac{1}{24}x^4.$$

(b) State Abel's Theorem. Show that

$$\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}\ldots\ldots\ldots\ldots -1\leq x\leq 1$$

and $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots\ldots\ldots.$

(1)

(c) Prove that for every continuous function f on [0, 1], the sequence of polynomials $B_n f \to f$ uniformly on [0, 1], where $(B_n f)$ is the sequence of Bernstein's polynomials for the function f.

7.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4102 **H**

Unique Paper Code : 2352012402

Name of the Paper : Multivariate Calculus

Name of the Course : **B.A./B.Sc. (H)**

Semester : IV (DSC)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Let f be the function defined by

$$f(x,y)=\begin{cases} \dfrac{xy^3}{x^2+y^6}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

Is f continuous at (0,0)? Explain.

(b) A cylindrical tank is 4 ft high and has an outer diameter of 2 ft. The walls of tank are 0.2 inches thick. Approximate the volume of interior of tank assuming the tank has a top and a bottom that are both 0.2 inches thick.

(c) Find an equation for each horizontal tangent plane to the surface

$$z = 5 - x^2 - y^2 + 4y.$$

2. (a) Let $f(x, y, z) = ye^{x+z} + ze^{y-x}$. At the point (2,2,–2) find the unit vector pointing in the direction of most rapid increase of f. And what is the value of most rapid increase of f?

(b) Find the absolute extrema of the function f(x, y) = 2 sin x + 5 cos y on the set S where S is the rectangular region with vertices (0,0), (2,0), (2,5) and (0,5).

(c) Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

3. (a) (i) Sketch the region of integration and compute the double integral

$$\int_0^{\pi/2} \int_0^{\sin x} e^x \cos x \, dydx .$$

(ii) Evaluate $\int\int_D (2y - x) dA$ where D is the region bounded by $y = x^2$, $y = 2x$.

(b) Evaluate the area bounded by $r = 2 \cos\theta$.

(c) Evaluate the given integral by converting to polar coordinates

$$I = \int_0^2 \int_y^{\sqrt{8-y^2}} \frac{1}{\sqrt{1+x^2+y^2}} dydx .$$

4. (a) (i) Set up a triple integral to find volume of solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

(ii) Set up a double integral to find volume of solid region bounded by ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 .$$

(b) Use cylindrical co-ordinates to compute integral $\iiint_D z(x^2 + y^2)^{-1/2} \, dxdydz$ where D is the solid bounded above by plane $z = 2$ and bounded below by surface $2z = x^2 + y^2$.

(c) Find volume of solid D where D is the intersection of solid sphere $x^2 + y^2 + z^2 \le 9$ and solid cylinder $x^2 + y^2 \le 1$.

5. (a) Evaluate $\oint_C (x^2 y dx - xy\, dy)$, where C is the path that begins at (0,0), goes to (1,1) along the parabola $y = x^2$, and then return to (0,0) along the line $y = x$.

(b) Prove or disprove that the line integrals are path independent.

(c) Evaluate the line integral

$$\oint_C [(2x - x^2 y)e^{-xy} + \tan^{-1} y]\boldsymbol{i} + [\tfrac{x}{y^2+1} - x^3 e^{-xy}]\boldsymbol{j}. dR,$$

where C, the curve with parametric equations $x = t^2 \cos \pi t$, $y = e^{-t} \sin \pi t$, $0 \le t \le 1$.

6. (a) Find the area enclosed by the semicircle $y = \sqrt{4 - x^2}$ using the line integral.

(b) Find the mass of the homogenous lamina of density $\delta = x$ that has the shape of the surface S given by $z = 4 - x - 2y$ with $z \ge 0$, $x \ge 0$ and $y \ge 0$.

(c) Let $F = 2xi - 3yj + 5zk$, and let S be the hemisphere $z = \sqrt{9 - x^2 - y^2}$ together with the disk $x^2 + y^2 \le 9$ in the xy–plane. Verify the divergence theorem.

9

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5873 **H**

Unique Paper Code : 2354002003

Name of the Paper : Elements of Real Analysis

Name of the Course : **COMMON PROG GROUP**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Simple Calculator allowed.

1. (a) Let $a < b$ in an ordered field F. Then prove that $b = \sup(a, b)$.

   (b) Prove that any complete ordered field is Archimedean.

(c) Prove that for all x, y $\in$ ordered field F, $\max\{x, y\} = \frac{x+y+|x-y|}{2}$ and $\min\{x,y\} = \frac{x+y-|x-y|}{2}$.

2. (a) Prove that in an ordered field F, u is a lower bound of a set A if and only if $-u$ is an upper bound of the set $-A = \{-a : a \in A\}$.

(b) Verily whether all the field axioms for the set $\mathbb{R}^2$ with $(x, y) + (u, v) = (x + u, y + v)$ and $(x, y).(u, v) = (xu, yv)$ are satisfied or not.

(c) Define a convergent sequence. Using the definition of convergence, show that

$$\lim_{n\to\infty} \frac{3n^2+n-5}{n^2+6n} = 3.$$

3. (a) Prove that every monotonically increasing bounded above sequence is convergent.

(b) Let $X = \langle x_n \rangle$ and $Y = \langle y_n \rangle$ be sequences of real numbers that converges to x and y, respectively, then prove that the sequence X – Y converges to x – y.

(c) Prove that the sequence $\langle x_n \rangle = \langle (-1)^n \rangle$ is not a cauchy sequence.

4. (a) Let $\langle x_n \rangle$ be a sequence of real numbers defined inductively by $x_1 = 1$ and $x_{n+1} = \sqrt{4x_n + 5}$, for all $n \in N$. Prove that $\langle x_n \rangle$ converges and find its limit.

(b) State second squeeze principle for convergence of sequences and use it to prove that $\lim_{n\to\infty} \frac{2n+3}{3n-7} = \frac{2}{3}$.

(c) Suppose that $\langle x_n \rangle$ is a sequence of real numbers for which there is a constant $c > 0$ such that for all $n \in N$, $|x_{n+1} - x_n| < \frac{C}{2^n}$. Then prove that $\langle x_n \rangle$ is a cauchy sequence.

5. (a) Show that the nth partial sum of series $\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots (a \neq 0)$ is $\frac{a}{1-r}\left(1-r^n\right)$, if $r \neq +1$. Hence, show that the geometric series converges to $\frac{a}{(1-r)}$ if $|r| < 1$.

(b) State and prove Cauchy Criterion for convergence of series.

(c) (i) Determine whether the geometric series

$$\sum_{n=1}^{\infty} \frac{3^n}{4^{n+2}}$$

converges or diverges. Find the sum of the series if it converges.

(ii) Show that 0.002002002 ..... converges and find it's sum.

6. (a) State Integral Test. Using this test or otherwise, test the convergence of the following series :

(i) $\sum_{n=1}^{\infty} \frac{n}{n^2+10}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(b) Test the convergence of following series :

(i) $\sum_{n=2}^{\infty} \frac{1}{n!}$

(ii) $\sum_{n=1}^{\infty} \left(\frac{1}{3}+\frac{1}{n}\right)^n$

(c) (i) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$ converges using Alternating Series Test.

(ii) Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$; for $p > 0$.

(c) Use the Runge-Kutta Method fourth-order, determine the approximate solution of the initial value Problem $\frac{dx}{dt} = \frac{x}{t}$, $x(0) = 1$, $1 \le t \le 2$, taking the step size as $h = 0.5$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4140 **H**

Unique Paper Code : 2352012403

Name of the Paper : NUMERICAL ANALYSIS

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of non-programmable Scientific Calculator is allowed.

P.T.O.

1. (a) Determine a formula which relates the number of iterations n, required by the bisection method to converge to within an absolute error tolerance ε, starting from the initial interval (a, b).

(b) Perform up to four iterations using the false position to approximate a root of $f(x) = e^x + x^2 - x - 4$ in the interval $(-2, -1)$.

(c) Verify that $x = \sqrt{a}$ is a fixed point of the function $g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$. Use the fixed point iteration scheme to determine the order of convergence and the asymptotic error constant of the sequence $p_n = g(p_{n-1})$ towards $x = \sqrt{a}$.

(c) Derive the formula for the Trapezoidal rule and hence evaluate the approximate value of the integral $\int_1^2 \frac{1}{x}dx$.

6. (a) Determine the values for the coefficients A_0, A_1, and A_2 so that the quadrature formula

$$I(f) = \int_{-1}^{1} f(x)\,dx = A_0 f\left(\frac{-1}{3}\right) + A_1 f\left(\frac{1}{3}\right) + A_2 f(1)$$

has degree of precision at least 2.

(b) Use the Euler Method, determine the approximate solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $x(1) = 1$, $1 \le t \le 2$, taking the step size as $h = 0.5$.

x	0	1	2	3	4
$f(x)$	1	2	5	10	16

Hence estimate the value of f(1.5), f(2.5) and f(3.5).

5. (a) Prove that $f'(x_0) \approx \frac{f(x_0+h)-f(x_0)}{h}$ and determine the approximate value of the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 1$, taking h = 1, 0.1, 0.01 and 0.001.

(b) Verify that the second-order forward difference approximation for the first derivative provides the exact value of the derivative, regardless of h, for the functions $f(x) = 1$, $f(x) = x$ and $f(x) = x^2$, but not for the function $f(x) = x^3$.

2. (a) Calculate the fourth approximation of the root of the function $f(x) = e^{-x} - x$ using the Newton's method with $p_0 = 0$.

(b) Use the Secant method to calculate the root of the function $f(x) = 1.05 - 1.04x + \ln x$ using $p_0 = 1.10$ and $p_1 = 1.15$ with an absolute tolerance of 10^{-6} as a stopping condition.

(c) Determine the order of convergence of the Newton's method to find a root p of a twice differentiable function f on the interval [a, b], provided $f'(p) \neq 0$.

3. (a) Solve the following system of equations by using the LU Decomposition method

$$x + 4y + 3z = -4,$$

$$2x + 7y + 9z = -10,$$

$$5x + 8y - 2z = 9.$$

(b) Starting with initial vector $(x, y, z, t) = (0, 0, 0, 0)$, perform three iterations of Gauss-Seidel method to solve the following system of equations

$$4x + y + z + t = -5,$$

$$x + 8y + 2z + 3t = 23,$$

$$x + 2y - 5z = 9,$$

$$-x + 2z + 4t = 4.$$

(c) Compute T_{jac} and T_{gs} for the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$.

Will the Jacobi and Gauss Seidel method converge for any choice of initial vector $x^{(0)}$? Justify your answer.

4. (a) Find the Lagrange interpolation polynomial for the data set (0,1), (1,3) and (3,55). Also estimate the value at $x = 2.5$.

(b) The population of a town during the last six censuses are given below. Estimate the increase in the population from 1846 to 1848 by using the Newton interpolation polynomial.

Year	1811	1821	1831	1841	1851	1861
Population (in thousands)	9	12	20	17	37	42

(c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the given data :

(b) Discuss the convergence and uniform convergence of the series of functions

$$\Sigma(x^n + 1)^{-1}, \ x \neq 0 \qquad (6.5)$$

(c) If f_n is continuous on $D \subseteq R$ to R for each $n \in N$ and Σf_n converges to f uniformly on D then show that f is continuous on D. (6.5)

6. (a) (i) Find the radius of convergence of the power series (3)

$$\sum_{n=1}^{\infty} \frac{n^n}{n!}$$

(ii) Define sinx as a power series and find its radius of convergence. (3)

(b) If the power series $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence R, then the power series

$\sum_{n=1}^{\infty} na_n x^{n-1}$ and $\sum_{n=0}^{\infty} \frac{a_n}{n+1} x^{n+1}$ also have radius of convergence R. (6)

(c) Suppose that $f(x) = \sum_{n=0}^{\infty} a_n x^n$ has radius of convergence $R > 0$. Then f is differentiable on $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} na_n x^{n-1} \quad \text{for } |x| < R. \qquad (6)$$

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3005 **H**

Unique Paper Code : 32351402

Name of the Paper : Riemann Integration and Series of Functions

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of calculator is not allowed.

1. (a) Find the upper and lower Darboux integral for $f(x) = 2x + 1$ and show that it is integrable on $[1,2]$. Hence show that $\int_1^2 (2x + 1)dx = 4.$ 6.5)

(b) Prove that a bounded function f on [a, b] is Riemann integrable if and only if it is Darboux integrable in which case the values of the integrals agree. (6.5)

(c) Prove that every continuous function f on [a, b] is integrable. (6.5)

2. (a) (i) State and prove intermediate value theorem for integrals.

(ii) Give example of a function f which is not integrable for which |f| is integrable. (6)

(b) If u and v are continuous functions on [a, b] that are differentiable on [a, b] and if u′ and v′ are integrable on [a, b] then prove

$$\int_a^b u(x)v'(x)+\int_a^b u'(x)v(x)=u(b)v(b)-u(a)v(a)$$

(6)

(c) Prove that if f is a piecewise continuous function or a bounded piecewise monotonic function on [a, b] then f is integrable on [a, b]. (6)

3. (a) Prove that $\int_0^\infty e^{-t}t^{s-1}dt$ is convergent if and only if $s > 0$. (6)

(b) (i) Prove that $\int_1^\infty \frac{\sin}{x^2}dx$ converges absolutely

(ii) Prove that $\int_1^\infty \frac{\sqrt{x}}{x^3+5}dx$ is convergent. (6)

(c) Test convergence of

(i) $\int_2^\infty \frac{2x^2}{x^4-1}dx$ (ii) $\int_0^\infty e^{-x^2}x^2\,dx$ (6)

4. (a) Let $\langle f_n \rangle$ be a sequence of bounded functions on $A \subseteq R$. Then this sequence converges uniformly on A to a bounded function f if and only if for each $\in > 0$ there is a number $H(\in)$ in N such that for all $m, n \geq H(\in)$ then $||f_m - f_n||_A \leq \in$. (6.5)

(b) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \geq 0$, $n \in N$. Show that the sequence $\langle f_n \rangle$ converges non-uniformly on $[0, \infty)$ and converges uniformly on $[a, \infty)$, $a > 0$. (6.5)

(c) If $a > 0$, show that $\lim_{n\to\infty}\left(\int_a^\pi \frac{\text{Sin } nx}{nx}dx\right)=0$. What happens if $a = 0$. (6.5)

5. (a) State and prove Weierstrass M-test for the uniform Convergence of a series of functions. (6.5)

(c) Let V and W be vector spaces. Let $T: V \to W$ be linear and let $\{w_1, w_2, \ldots, w_k\}$ be a linearly independent subset of Range of T. Prove that if $S = \{v_1, v_2, \ldots v_k\}$ is chosen so that $T(v_i) = w_i$ for $i = 1, \ldots, k$. Then S is linearly independent. (6.5)

6. (a) Let T be a linear operator on a finite dimensional vector space V. Let β and β' be the ordered basis for V. Suppose that Q is the change of coordinate matrix that changes β' coordinates into β coordinates, then $[T]_{\beta'} = Q^{-1}[T]_\beta Q$. (6.5)

(b) Let β, γ be the standard ordered basis of $P_1(\mathbb{R})$ and $\mathbb{R}^2$ respectively.

Let $T: P_1(\mathbb{R}) \to \mathbb{R}^2$ be a linear transformation defined by $T(a + bx) = (a, a+b)$.

Find $[T]_\beta^\gamma$, $\left[T^{-1}\right]_\gamma^\beta$ and verify that $\left[T^{-1}\right]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$. (6.5)

(c) Let $U: P_3(\mathbb{R}) \to P_2(\mathbb{R})$ and $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear transformations respectively defined by $U(f(x)) = f'(x)$ and $T(f(x)) = \int_0^x f(t)\,dt$. Prove that $[UT]_\beta = [U]_\alpha^\beta [T]_\beta^\alpha$ where α and β are standard ordered basis of $P_3(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. (6.5)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3116 **H**

Unique Paper Code : 32351403

Name of the Paper : Ring Theory & Linear Algebra-I

Name of the Course : **B.Sc. (Hons.) Mathematics CBCS (LOCF)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Prove that every finite Integral domain is a field. Give an example of an infinite integral domain which is not a field, Justify. (6)

(b) (i) Let F be a field of order 2^n. Prove that the characteristic of F is 2.

(ii) Find all the units in $\mathbb{Z}[i]$ (6)

(c) Prove that the set of all the nilpotent elements of a commutative ring form a subring. (6)

2. (a) Let R be a commutative ring with unity and A be an ideal of R then prove that R/A is an integral domain if and only if A is a prime ideal of R. (6)

(b) Prove that $\mathbb{Z}[i]/\langle 1-i \rangle$ is a field. (6)

(c) Find all the maximal ideals of $\mathbb{Z}_{20}$. (6)

3. (a) Find all the ring homomorphisms from $\mathbb{Z}_4$ to $\mathbb{Z}_{10}$. (6.5)

(b) Let $R = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| a,b,c \in \mathbb{Z} \right\}$ and Φ be the mapping that takes $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ to a. Show that

(i) Φ is a ring homomorphism

(ii) Determine the kernel of Φ.

(iii) Is Φ a one-one mapping. Justify. (6.5)

(c) State and prove first isomorphism theorem for rings. (6.5)

4. (a) Let V(F) be a vector space.

(i) Prove that the intersection of two subspaces of V(F) is also a subspace of V(F).

(ii) Show that union of two subspaces of V(F) may not be a subspace of V(F). Discuss the condition under which union of two subspaces will also form a subspace of V(F). (6)

(b) Let S be a linearly independent subset of a vector space V(F), and let v be a vector in V that is not in S. Then $S \cup \{v\}$ is linearly dependent iff $v \in \text{Span}(S)$. (6)

(c) Let u, v, w be distinct vectors of a vector space V. Show that if {u,v,w} is a basis for V, then {u+v+w, v+w, w} is also a basis for V. (6)

5. (a) Let V(F) and W(F) be vector spaces and let $T: V \to W$ be a linear transformation. If V is a finite-dimensional, then

$$\text{Dim}(V) = \text{Nullity}(T) + \text{Rank}(T). \quad (6.5)$$

(b) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ and $U: \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformations respectively defined by

$T(a_1, a_2) = (a_1 + 3a_2, 0, 2a_1 - 4a_2)$

$U(a_1, a_2) = (a_1 - a_2, 2a_1, 3a_1 + 2a_2)$

Let β, γ be the standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively, Prove that

(i) $[T+U]_\beta^\gamma + [T]_\beta^\gamma + [U]_\beta^\gamma$

(ii) $[aT]_\beta^\gamma = a[T]_\beta^\gamma$ for all scalars a. (6.5)

(c) Find the payoff from a bull spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3071 H

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (Hons) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

P.T.O.

1. (a) Define Convexity of a bond and find the relation between the Convexity, Duration and Bond price. How does convexity measure sensitivity of the portfolio?

(b) Consider the three bonds having payments as shown in the table below. They are traded to procure a 12% yield with continuous compounding.

End of year payments	Bond A	Bond B	Bond C
Year 1	1000	500	0
Year 2	1000	500	0
Year 3	1000+10000	500+10000	0+10000

Determine the price and duration of each bond.

(c) Explain Forward Rates. Derive the following relation :

$$R_F = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where R_F is the forward interest rate for the period between T_1 and T_2. R_1 and R_2 are the zero rates for maturities T_1 and T_2 respectively. What happens when $R_2 > R_1$?

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

(b) Companies A and B have been offered the following rates per annum on a ₹10 million loan for 5 years :

	Fixed rate	Floating rate
Company A	12.0%	LIBOR+ 0.1%
Company B	14.5%	LIBOR + 0.9%

Company A requires a floating-rate loan; Company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right],$$

where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(S_T > K)$, the probability that a European call option be exercised in a risk-neutral world.

(b) Show that the Black—Scholes-Merton formulas for call and put options satisfy the put-call parity.

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹52, the strike price is ₹50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is three months?

(You can use values: $\ln(26/25) = 0.0392$, $\exp(-0.03) = 0.9704$)

2. (a) (i) An investor sells a European call option with strike price of K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

(ii) An investor sells a European call on a share for ₹6, the stock price is ₹45 and the strike price is ₹52. Under what circumstances will the seller of the option make a profit? Under what circumstances will the option be exercised?

(b) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(c) Explain Hedging. A United States company expects to pay 1 million Euros in 3 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the gamma of a European call and the gamma of a European put on a non-dividend-paying stock.

(c) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments, ($e^{-0.005} = 0.9950$).

4. (a) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ₹38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and A shares of the stock. What is the value of A which makes the

portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0.005} = 1.005$).

(b) A stock price is currently ₹50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹51? (You can use exponential value: $e^{-0.0125} = 0.9876$).

(c) Consider a two-period binomial model with current stock price S_0 = ₹100, the up factor $u = 1.2$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹104 and maturity $T = 1$ year. ($e^{-0.025} = 0.9753$)

M

06

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2933 **H**

Unique Paper Code : 32351601

Name of the Paper : Complex Analysis (including Practicals)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Each question consists of **three** parts. Attempt any **two** parts from each question.

1. (a) (i) Find and sketch, showing corresponding orientations, the image of the hyperbola $x^2 - y^2 = c_1$ $(c_1 < 0)$ under the transformation $w = z^2$.

(ii) Suppose $(z)=\begin{cases} z/|z| & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. Check the continuity of f at $z = 0$. Justify your answer.

(4+2=6)

P.T.O.

(b) (i) Determine the points where $f(z) = (x^3 + 3xy^2 - x) + i(y^3 + 3x^2y - y)$ is differentiable. Is f analytic at those points? Justify your answer.

(ii) Suppose f(z) is analytic. Can $g(z)=\overline{f(z)}$ be analytic? Justify your answer. (4+2=6)

(c) (i) Suppose $f(z) = e^{-x}e^{-iy}$. Show that $f'(z)$ and $f''(z)$ exist everywhere. Hence prove that $f''(z) = f(z)$.

(ii) Show that $\log(i^{1/2}) = (1/2) \log i$.

(3.5+2.5=6)

2. (a) (i) Find all the roots of the equation $\sin z = 5$.

(ii) Show that $|\exp(z^2)| \leq \exp(|z^2|)$. (4+2=6)

(b) (i) Show that $\sin \bar{z}$ is not analytic function anywhere.

(ii) Compute $\lim_{z\to\infty} \frac{2z^3 - 1}{z^2 + 1}$. (5+1=6)

(c) Evaluate $\int_C \frac{dz}{z}$, where C is a positively oriented circle $z = 2e^{i\theta}$ $(-\pi \leq \theta \leq \pi)$. (6)

3. (a) Let $f(z) = \pi \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1 + i and i; orientation of C being in the anticlockwise direction. Parametrize the curve C and evaluate $\int_C f(z)dz$. (6)

(b) Let C denote a contour of length L and suppose that a function f(z) is piecewise continuous on C. If M is a non-negative constant such that $|f(z)| \leq M$ for all points z on C at which f(z) is defined, then prove that $\left|\int_C f(z)dz\right| \leq ML$. (6)

(c) If a function f is analytic at a given point, then prove that its derivatives of all orders are analytic there too. (6)

4. (a) State Cauchy integral formula and its extension. Evaluate $\oint_C \frac{z+1}{z^4 + 2iz^3} dz$, where C is the circle $|z| = 1$. (6)

(b) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z)}{z - z_0} dz = \int_C \frac{f(z)}{(z - z_0)^2} dz$$ (6)

(c) State and prove Fundamental theorem of Algebra. (6)

5. (a) Find limit of the following sequences as $n \to \infty$:

(i) $z_n = \frac{1}{n^3} + i$ (n = 1,2,....)

(ii) $w_n = -2 + i\frac{(-1)^n}{n^2}$ (n = 1,2,...) (3+3.5)

(b) Represent the function $f(z)=\frac{z+1}{z-1}$.

(i) by its Maclaurin series and state the domain where the representation is valid.

(ii) by its Laurent series in the domain $1 < |z| < \infty$. (3+3.5)

(c) Find the residue at $z = 0$ of the functions :

(i) $\frac{1}{z+z^2}$ (ii) $z\cos(1/z)$ (3+3.5)

6. (a) State Cauchy Residue Theorem and use it to evaluate the integral of $\frac{1}{1+z^2}$ around the circle $|z| = 2$ in the positive sense. (7)

(b) Show that the point $z = 0$ is a pole of the function $z(e^z - 1)$. Also find order of the pole and corresponding residue. (7)

(c) In each case, write the principal part of the function at its isolated singular point and determine the type of singular point with full explanation :

(i) $e^{1/z}$ (ii) $\frac{z^2}{1+z}$ (3.5+3.5)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2987 **H**

Unique Paper Code : 32351602

Name of the Paper : Ring Theory and Linear Algebra-II

Name of the Course : **B.Sc. (H) Mathematics (CBCS-LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) State and prove the Division Algorithm for $\mathbb{F}[x]$, where $\mathbb{F}$ is a field.

(b) State and prove Eisenstein's Criterion.

(c) Let $\mathbb{F}$ be a field and

$I = \{a_0 + a_1x + a_2x^2 + \cdots + a_nx^n | a_i \in \mathbb{F},$

$i = 0,1,2,\ldots, n$ and $\sum_{i=0}^{n} a_i = 0\}$. Show that I is an ideal of $\mathbb{F}[x]$ and find a generator of I.

(6,6,6)

2. (a) Let $\mathbb{F}$ be a field and $p(x) \in \mathbb{F}[x]$. Prove that $\langle p(x)\rangle$, is a maximal ideal in $\mathbb{F}[x]$ if and only if $p(x)$ is irreducible.

(b) Prove that every Euclidean Domain is a Principal Ideal Domain.

(c) In the ring $\mathbb{Z}[\sqrt{5}]$, show that the element $1+\sqrt{5}$ is irreducible but not prime. (6.5,6.5,6.5)

3. (a) Let $V = \mathbb{R}^3$ and define $f_1, f_2, f_3 \in V^*$ as follows :

$$f_1(x,y,z) = x - \frac{1}{2}y, \quad f_2(x,y,z) = \frac{1}{2}y, \quad \&$$

$$f_3(x,y,z) = -x + z$$

Prove that $\{f_1, f_2, f_3\}$ is a basis for V^* and then find a basis for V for which it is the dual basis.

(b) Let $V = P_3(\mathbb{R})$ and $T(f(x)) = f(x) + f(2)x$. Show that T is a linear operator on V. Further, find the eigenvalues of T and an ordered basis β for V such that $[T]_\beta$ is a diagonal matrix.

(c) Test the linear operator $T: \mathbb{C}^2 \to \mathbb{C}^2$, $T(z, w) = (z + iw, iz + w)$ for diagonalizability and if diagonalizable, find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (6,6,6)

4. (a) Let T be a linear operator on a finite dimensional vector space V and let W be a T-invariant subspace of V. State relationship between the characteristic polynomial of T_W and characteristic polynomial of T. Verify that relationship for the linear operator $T: \mathbb{R}^4 \to \mathbb{R}^4$, $T(a, b, c, d) = (a + b + 2c - d, b + d, 2c - d, c + d)$ and the T-invariant subspace $W = \{(t, s, 0, 0): t, s \in \mathbb{R}\}$ of V.

(b) State Cayley-Hamilton theorem for an n-dimensional vector space V and use it to prove Cayley-Hamilton theorem for matrices.

(c) Let T be a linear operator on a vector space Land let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be distinct eigenvalues of T. For each $i = 1,2,\ldots, k$, let S_i be a finite linearly independent subset of the eigenspace E_{λ_i}. Then prove that $S = S_1 \cup S_2 \cup \cdots \cup S_k$ is a linearly independent subset of V. (6.5,6.5,6.5)

5. (a) (i) Let $V = M_{2\times2}(\mathbb{C})$ together with the Frobenius inner product given by

$$\langle A, B\rangle = tr(B^*A) \text{ for all } A, B \in V. \text{ Let}$$

$$A = \begin{bmatrix} 1 & 2+i \\ 3 & i \end{bmatrix} \text{ and } A = \begin{bmatrix} 1+i & 0 \\ i & -i \end{bmatrix}. \text{ Compute}$$

$\langle A, B\rangle$, $||A||$, $||B||$ and $||A + B||$. Then, verify both the Cauchy-Schwarz inequality and the triangle inequality.

(ii) Provide a reason why $\langle(a, b), (c, d)\rangle = ac - bd$ is not an inner product on $\mathbb{R}^2$.

(b) Apply the Gram-Schmidt process to a subset $S = \{\sin t, \cos t, 1, t\}$ of the inner product space $V = \text{span}(S)$ with the inner product

$\langle f, g\rangle = \int_0^{\pi} f(t)g(t)dt$ to obtain an orthogonal basis for V. Then, normalize the vectors in this basis to obtain an orthonormal basis β for V and compute the Fourier coefficients of the vector h(t) = 2t + 1 relative to β.

(c) Prove that an orthogonal subset of a finite-dimensional inner product space V can be extended to an orthonormal basis for V. Hence, or otherwise, prove that for any subspace W of a finite-dimensional inner product space V, $\dim(V) = \dim(W) + \dim(W^{\perp})$.

(4+2,6,6)

6. (a) Let T be a linear operator on a complex finite-dimensional inner product space V with an adjoint T*. Prove that

(i) T is self-adjoint if and only if $\langle Tx, x\rangle$ is real for all $x \in V$.

(ii) If $\langle Tx, x\rangle = 0$ for all $x \in V$, then $T = T_0$, the zero operator on V.

(b) (i) Show that the reflection operator on $\mathbb{R}^2$ about a line through the origin is an orthogonal operator.

(ii) Show that the pair of matrices $A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{pmatrix}$ are unitarily equivalent.

(c) (i) For the set of data S = {(−3,9), (−2,6), (0,2), (1,1)} use the least square approximation to find the line of best fit.

(ii) For $V = \mathbb{R}^2$ with the standard inner product and a linear operator $T: V \to V$, given by $T(a, b) = (2a + b, a - 3b)$ for all $a, b \in V$, evaluate $T^*(3,5)$.

(4+2.5,3+3.5,4+2.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3183 **H**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each questions.
3. **All** questions carry equal marks.

1. (a) Define a Convex Set. Show that the set S defined as :

$$S = \{(x, y) \mid y^2 \leq 4x\} \text{ is a Convex Set.}$$

(b) Let $x_1 = 1, x_2 = 2, x_3 = 4$ be a feasible solution to the system of equations :

$$2x_1 + 3x_2 - x_3 = 4$$

$$3x_1 - x_2 + x_3 = 5$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Consider the following linear programming problem:

Minimize $z = cx$

subject to $Ax = b,\ x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution with objective function value z_B corresponding to a basis B where $x_B = B^{-1}b$. By entering an a_j with $z_j - c_j > 0$ and removing a b_r subject to :

$$\frac{x_{Br}}{y_{rj}} = \text{Min}\left[\frac{x_{Bi}}{y_{ij}} : y_{ij} > 0\right]$$

Show that we can get a new feasible solution with improved value of the objective function compared to z_B.

2. (a) Using Simplex method, find the solution of the following linear programming problem :

Maximize $z = x_1 - 2x_2 + x_3$

subject to $x_1 + 2x_2 + x_3 \leq 12$

$2x_1 + x_2 - x_3 \leq 6$

$x_1 - 3x_2 \geq -9$

$x_1, x_2, x_3 \geq 0.$

(b) Using two phase method, solve the linear programming problem :

Minimize $z = -3x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0.$

(c) Solve the following linear programming problem by Big - M method :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0.$

3. (a) Consider the following primal problem (P) and dual problem (D) :

(P) Minimize $z = cx$

Subject to $Ax \geq b,\ x \geq 0$

(D) Maximize $z = wb$

Subject to $wA \leq c,\ w \geq 0,$

If x_0 (w_0) is an optimal solution to the primal (dual) problem then there exists a feasible solution $w_0(x_0)$ to the dual (primal) such that $cx_0 = w_0 b$.

(b) Use graphical method to solve the dual of the following linear programming problem :

Minimize $z = 6x_1 + 8x_2 + 7x_3 + 15x_4$

Subject to $x_1 + x_3 + 3x_4 \geq 4$

$x_2 + x_3 + x_4 \geq 3$

$x_1, x_2, x_3, x_4 \geq 0$

Further, find an optimal solution to the given problem from optimal solution of the dual problem.

(c) Obtain the dual of the following linear programming problem :

Maximize $z = 10x_1 + x_2 + 2x_3$

Subject to $x_1 + x_2 - 2x_3 \geq 10$

$x_1 + 4x_2 - 3x_3 = 3$

$4x_1 + x_2 + x_3 \leq 20$

$x_1 \geq,\ x_2 \leq 0$, and x_3 unrestricted in sign.

(a) A Company has four warehouses, a, b, c and d. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following amounts in stock.

Warehouse :	a	b	c	d
No. of units :	15	16	12	13

and the customer's requirements are

Customers :	A	B	C
No. of units	18	20	18

The table below shows the costs of transporting one unit from warehouses to the customers :

	a	b	c	d
A	8	9	6	3
B	6	11	5	10
C	3	8	7	9

Find the optimal schedule and minimum total transport cost.

(b) A Company is faced with the problem of assigning six different machines to six different jobs. Determine the optimal solution of the Assignment Problem with the following cost matrix :

	a	b	c	d	e	f
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	3	56	53	31	17	28

(c) For the following cost minimization Transportation Problem, find initial basic feasible solution by using North-West comer rule, Least Cost method and Vogel's approximation method. Compare the three solutions (in terms of cost).

	A	B	C	D	Supply
I	19	14	23	11	11
II	15	16	12	21	13
III	30	25	16	39	19
Demand	6	10	12	15	

(a) Define the Saddle point. The pay-off matrix of a game is given below. Find the best strategy for each player, and the value of a play of the game of A and B.

Player B

yer A

	I	II	III	IV	V
I	9	3	1	8	0
II	6	5	4	6	7
III	2	4	3	3	8
IV	5	6	2	2	1

(b) Convert the following Game problem into a linear programming problem for Player A and Player B and solve it by Simplex method.

	Player B		
Player A	3	-2	4
	-1	4	2

(c) Using Simplex method, solve the system of equations :

$$3x_1 + x_2 = 7$$

$$x_1 + x_2 = 3$$

Also write the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

4

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3472 **H**

Unique Paper Code : 42357618

Name of the Paper : Numerical Methods

Name of the Course : **B.Sc. (Prog.) Physical Sciences / Mathematical Sciences**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

P.T.O.

1. (a) Define the local truncation error. Round off the number 754682 to four significant digits and then calculate the absolute, relative and percentage errors. (6.25)

(b) Using the Regula Falsi method, compute the real root of the equation $x^3 = 6$ correct to four decimal places. (6.25)

(c) Perform four iterations of the Bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - 4x + 1 = 0$. (6.25)

2. (a) Compute $\sqrt{5}$ correct to four decimal places by using Newton-Raphson Method. (6.25)

(b) Solve the linear system $AX = b$, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ 3 & 14 & 28 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -2 \\ -8 \end{bmatrix}$$

by Gauss Elimination method using partial pivoting. (6.25)

(c) Using Secant method, find the smallest positive root of the equation $x^4 - x = 10$ correct to three decimal digits. (6.25)

3. (a) Perform three iterations of Gauss-Seidel method to solve the linear system

$$2x - y + 0z = 7$$

$$-x + 2y - z = 1$$

$$0x - y + 2z = 1$$

Take the initial approximation as $(x, y, z) = (0, 0, 0)$. (6.25)

(b) Find the inverse of the matrix A using Gauss-Jordan method, where

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

(6.25)

(c) Consider the following table :

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Use Newton divided difference formula to calculate the interpolating polynomial and give an estimate for f(0.25).

(6.25)

4. (a) Obtain the piecewise quadratic interpolating polynomial for

x	-2	-1	1	2	4
$f(x)$	-29	-8	-2	-5	7

Interpolate at x = 3.0.

(6.25)

(b) Construct the interpolating polynomial by using Gregory-Newton backward difference interpolation formula for the given data :

x	1	1.5	2.0	2.5
$f(x)$	2.7183	4.4817	7.3891	12.1825

Estimate the value of f(2.25).

(6.25)

(c) Construct the Richardson extrapolation table to find the derivative of $f(x) = 3^x$ at x = 3 using the central difference formula

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

taking h = 1,2.

(6.25)

5. (a) Consider the following table :

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

Find $f'(3)$ using

(i) Central difference formula

(ii) Three-point forward difference formula. (6.25)

(b) Compute the value of $\int_{-3}^{3} x^4\, dx$ using the Simpson's 1/3 rule taking six equal subintervals. (6.25)

(c) Apply trapezoidal rule to evaluate the integral

$$\int_0^6 \frac{dx}{1+x^2}$$

Calculate the difference between the exact value and the approximate value. (6.25)

6. (a) Apply Midpoint method to find approximate solution of the initial value problem

$$\frac{dy}{dx} = x + 2y, \quad y(0) = 0, \quad h = 0.1. \qquad (6.25)$$

(b) Apply Euler's method to find approximate solution of the initial value problem

$$\frac{dy}{dx} = \frac{x-y}{z}, \quad y(0) = 1, \quad 0 \le x \le 3, \quad h = 0.5. \qquad (6.25)$$

(c) Find an approximation to y(0.2), for the initial value problem

$$\frac{dy}{dx} = y + x, \quad y(0) = 1, \quad 0 \le x \le 0.2.$$

using the Heun's method with $h = 0.1$. Compare with exact solution. (6.25)

(c) Suppose the number of items produced in a factory during a week is a random variable with mean 500.

(i) What can be said about the probability that this week's production will be at least 1000?

(ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3473 **H**

Unique Paper Code : 42357602

Name of the Paper : DSE – Probability and Statistics

Name of the Course : **CBCS-LOCF: B.Sc. Physical Sciences / Mathematical Sciences**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all the **six** questions.
3. Each question has **three** parts. Attempt any **two** parts from each question.
4. Each part in Question **1, 3, 5** carries **6** marks.
5. Each part in Question **2, 4, 6** carries **6.5** marks.
6. Use of scientific calculator is allowed.

1. (a) Let $\{C_n\}$ be an arbitrary sequence of events. Show that

$$P\left(\bigcup_{n=1}^{\infty} C_n\right) \le \sum_{n=1}^{\infty} P(C_n).$$

(b) Find a formula for the probability distribution of the total number of heads obtained in six tosses of a balanced coin.

(c) The pdf of the random variable X is given by

$$f(x)=\begin{cases}\dfrac{c}{\sqrt{x}} & ; \ 0<x<4 \\ 0 & ; \ \text{elsewhere}\end{cases}$$

Find :

(i) The value of c.

(ii) $P\left(X<\frac{1}{4}\right)$ and $P(X>1)$.

(c) Using method of least square to fit a straight line for the following data:

X	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

6. (a) Let X and Y have joint probability mass function described as follows :

(x,y)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$P(x,y)$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

Find the correlation coefficient between X and Y.

(b) A soft drink vending machine is set so that the amount of drink dispensed is a random variable with a mean of 200 milliliters and a standard deviation of 15 milliliters. What is the probability that the average amount dispensed in a random sample of size 36 is at least 204 milliliters?

(c) Let X and Y have the joint pdf :

$$f(x,y)=\begin{cases}6y & ; \ 0<y<x<1\\ 0 & ; \ \text{elsewhere}\end{cases}$$

Show that E[Y] = E[E[Y|X]].

5. (a) Let

$$P(x,y)=\begin{cases}\dfrac{1}{16}, & \text{if } x=1,2,3,4 : y=1,2,3,4\\ 0, & \text{otherwise}\end{cases}$$

be the joint probability mass function of X and Y. Show that X and Y are independent.

(b) The joint density of two random variables X and Y is given as

$$f(x,y)=\begin{cases}x.\,e^{-x(1+y)}; & x>0 \text{ and } y>0\\ 0 & ; \text{elsewhere}\end{cases}$$

Find the regression equation of Y on X.

2. (a) Find the moment generating function of the random variable whose probability density is given by

$$f(x)=\begin{cases}e^{-x} & ; \ x>0\\ 0 & ; \ \text{elsewhere}\end{cases}$$

Use it to find an expression for μ_r', where $\mu_r' = E[X^r]$ is the rth moment about the origin.

(b) Define geometric distribution. If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?

(c) Show that Poisson distribution is a limiting case of binomial distribution when $n \to \infty$, $\theta \to 0$, while $n\theta = \lambda$ remains a constant.

3. (a) Show that if a random variable has a uniform density with the parameters α and β, the probability that it will take on a value less than $\alpha + p(\beta - \alpha)$ is equal to p.

(b) Let X and Y be two random variables with joint probability mass function :

$P(x,y)=\frac{x+y}{12}$; for x = 1, 2 and y = 1, 2, and zero elsewhere.

Show that $E[XY] \neq E[X]E[Y]$.

(c) Let X and Y be discrete random variables having joint probability mass function-

$$P(x,y)=\begin{cases}\left(\frac{1}{2}\right)^{x+y} & ; 1 \leq x < \infty,\ 1 \leq y < \infty,\ x, y \in Z \\ 0 & ; \text{elsewhere}\end{cases}$$

Determine the joint moment generating function of X and Y.

4. (a) Let X_1 and X_2 have the joint pdf:

$$f(x_1,x_2)=\begin{cases}x_1+x_2 & ; 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0 & ; \quad \text{otherwise}\end{cases}$$

(i) Find the marginal pdf of X_1, X_2.

(ii) Find the probability $P\left(X_1 \leq \frac{1}{2}\right)$.

(iii) Find the probability $P(X_1 + X_2 \leq 1)$.

(b) Let (X, Y) be a discrete random variable having joint probability mass function :

$$P(x,y)=\begin{cases}\frac{x+2y}{18} & ; (x,y)=(1,1),(1,2),(2,1),(2,2) \\ 0 & ; \text{elsewhere}\end{cases}$$

Determine the conditional mean and variance of Y given $X = x$ for $x = 1$.