Sketching Hyperbola :

→ Hyperbola with centre (h,k) and focal axis parallel to x-axis

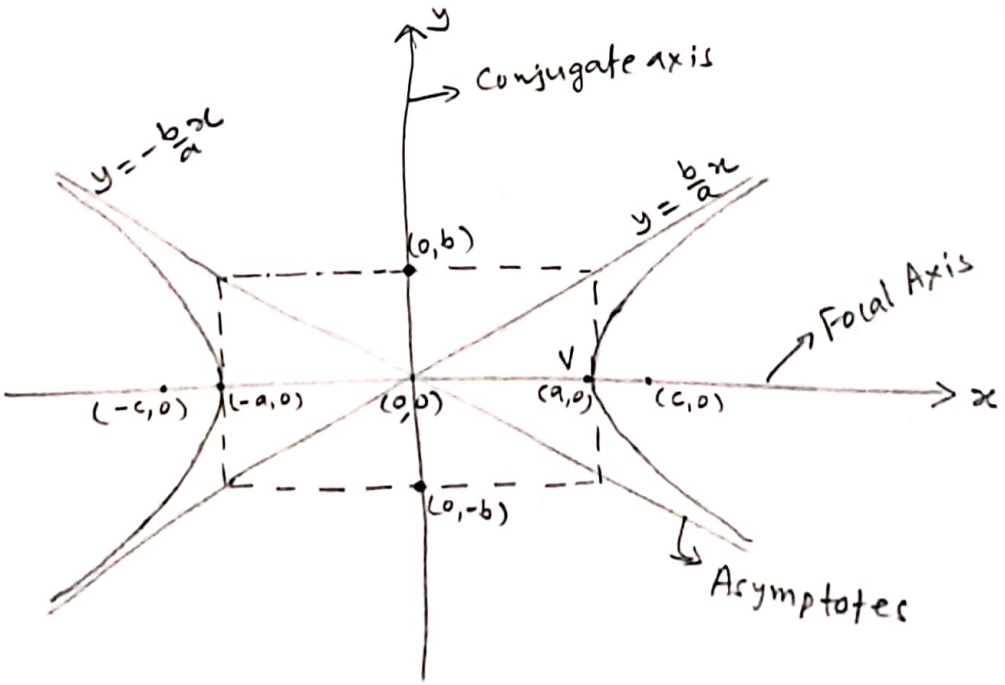
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

→ Hyperbola with centre (h,k) and focal axis parallel to y-axis

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

→ Eqn. of ~~parabola~~ Hyperbola with centre $(0,0)$ and focal axis parallel to x-axis

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$



eqn. of asymptotes are $\boxed{y = \pm \frac{b}{a}x.}$

P.T.O.

The number a is called the semifocal axis of the hyperbola.

$b \rightarrow$ semiconjugate axis.

$c = \sqrt{a^2+b^2}$

Steps to sketch hyperbolas:

Step1: Determine whether the focal axis is on the x-axis or the y-axis. This can be ascertained from the location of the minus sign in the eqn. The focal axis is along the x-axis when the minus sign precedes the y^2- term and it is along the y-axis when the minus sign precedes the x^2- term.

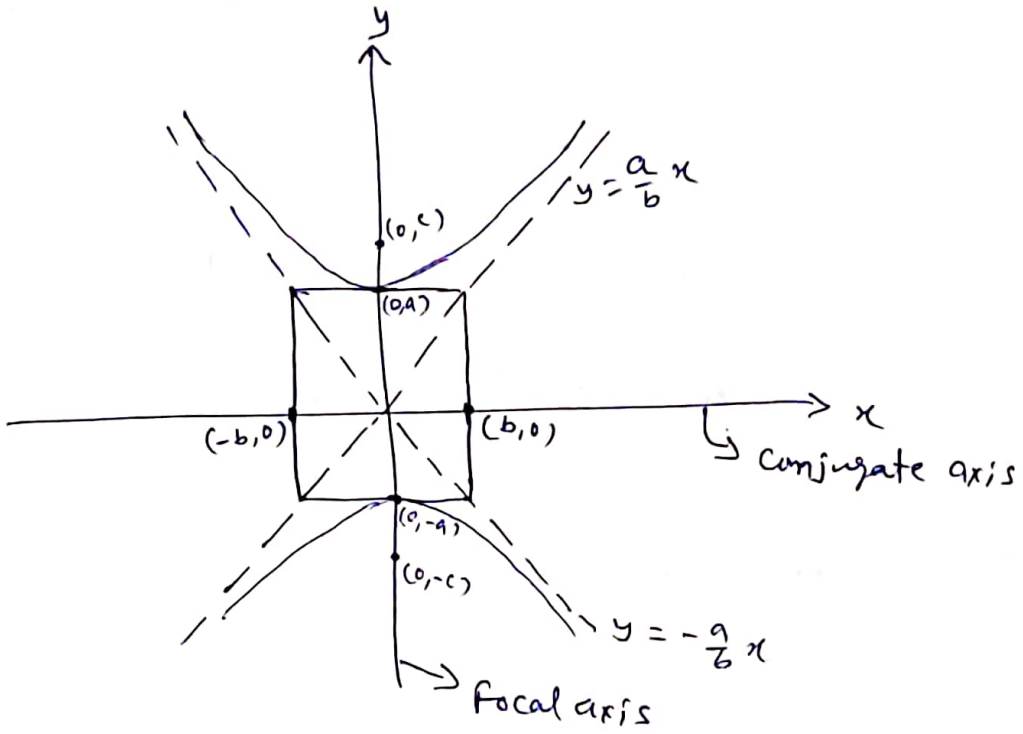
Step2: Determine the values of a and b and draw a box extending a units on either side of the centre along the focal axis and b units on either side of the center along the conjugate axis.

Step3: Draw the asymptotes along the diagonals of the box.

Step4: Using the box and the asymptotes as a tool, sketch the graph of the hyperbola.

[3] Eqn. of hyperbola with centre (0,0) and focal axis parallel to y-axis is

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

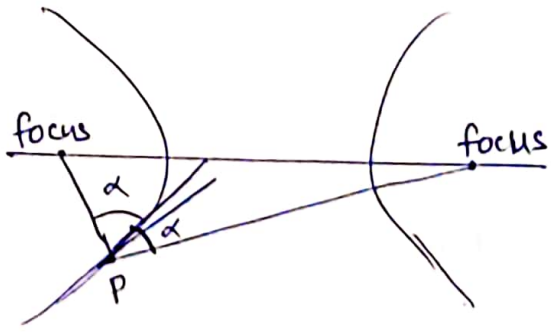


$$c = \sqrt{a^2+b^2}$$.

□

Reflection property of Hyperbola:

A line tangent to ~~tangent to~~ a hyperbola at a point P makes equal angles with the lines joining P to the foci.



Que.ⁿ 20(b) Sketch the hyperbola, and Label the vertices, foci, and asymptotes (P-762)

$$4y^2 - x^2 + 40y - 4x = -60 \quad \text{—(1.)}$$

Sol^{n}:

$$4(y^2+10y) - (x^2+4x) = -60$$

$$\Rightarrow 4(y^2+10y+25) - (x^2+4x+4) = -60+100-4$$

$$\Rightarrow 4(y+5)^2 - (x+2)^2 = 36$$

$$\Rightarrow \frac{(y+5)^2}{9} - \frac{(x+2)^2}{36} = 1 \quad \text{—(2.)}$$

Compare (2.) with $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ —(3.)

$\therefore$ The centre (h,k) is $(-2,-5)$.

Semifocal axis $\boxed{a = 3}$ and y-axis is focal axis.

Semiconjugate axis $\boxed{b = 6}$ and x-axis is conjugate axis.

For asymptotes: $\frac{(y+5)^2}{9} - \frac{(x+2)^2}{36} = 0$

$$\Rightarrow y+5 = \pm\frac{6}{3}(x+2)$$

$$\Rightarrow \boxed{y = 2x-1} \text{ and } \boxed{y = -2x-9}$$

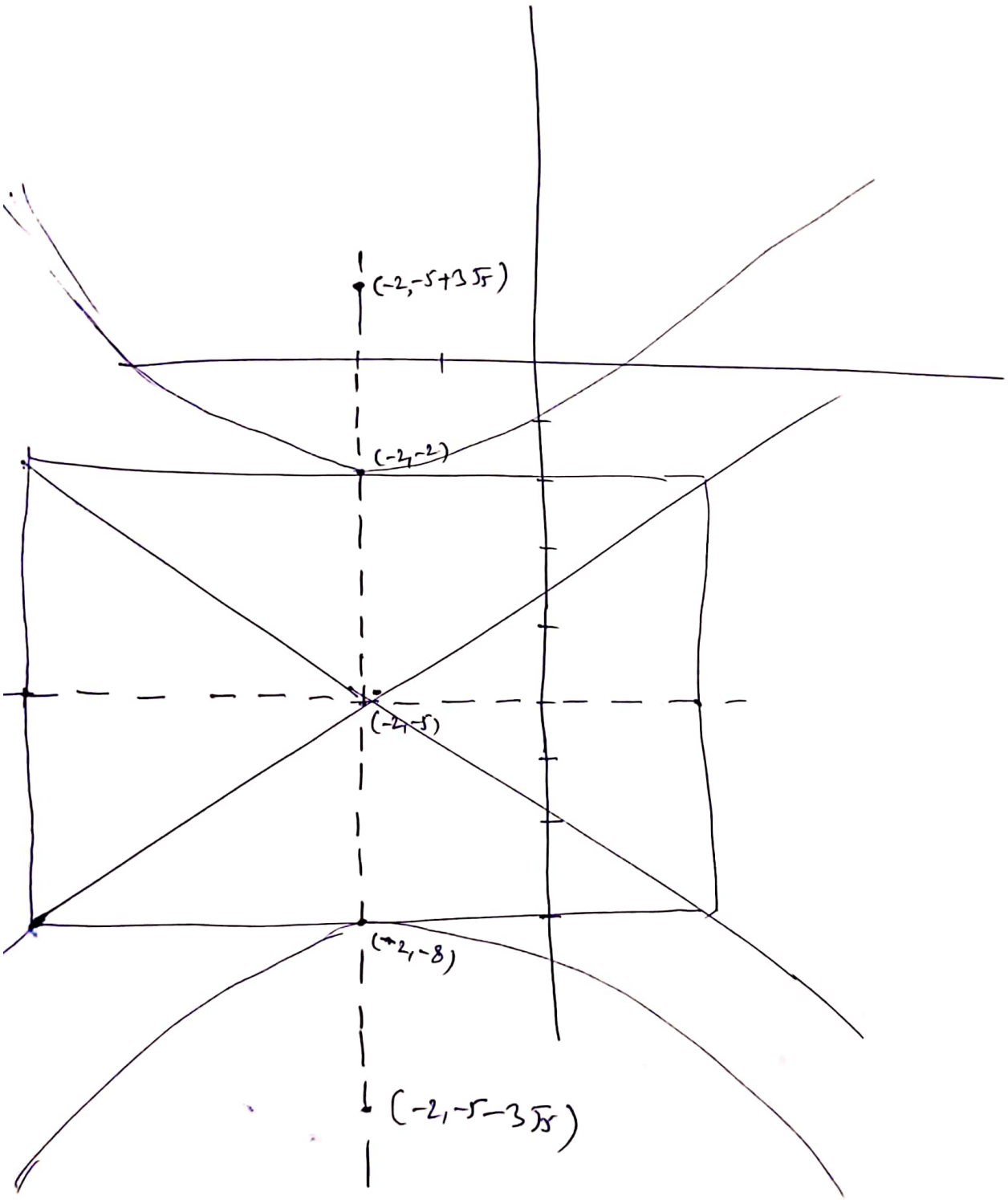
Now $c = \sqrt{a^2+b^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

Centre: $(-2,-5)$

Vertices: $(-2,-2)$ and $(-2,-8)$

Foci: At a distance c from centeron focalaxis, $(-2,-5+3\sqrt{5})$ and $(-2,-5-3\sqrt{5})$.

ctd... P.T.O



□

Que: 38(a) (P-765) Find an eqn. for a hyperbola with foci (1, 8) and (1, -12); vertices 4 units apart.

Soln. It is clear from the coordinates of foci that the focal axis of the hyperbola is y-axis.

The centre is the midpoint of two foci. Therefore coordinates of centre is (1, -2).

Given that vertices are 4 units apart to each other. As we know vertices lies on the focal axis and equidistant from centre. Here centre is (1, -2).

Therefore keeping in mind the position of centre (1, -2), the coordinates of vertices are (1, 0) & (1, -4).

∴ here $a = 2$, $c = 10$, $b = ?$

Since $c = \sqrt{a^2 + b^2} \Rightarrow 10 = \sqrt{(2)^2 + b^2}$

$\Rightarrow b = \sqrt{96} = 4\sqrt{6}$

∴ Eqn. of hyperbola is

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\boxed{\frac{(y+2)^2}{4} - \frac{(x-1)^2}{96} = 1}$$

□