## First Derivative Test:

Suppose $f$ is continuous at a critical point $x_0$.

(a.) If $f'(x) > 0$ on an open interval extending left from $x_0$ and $f'(x) < 0$ on an open interval extending right from $x_0$, then $f$ has a relative maximum at $x_0$.

(b) If $f'(x) < 0$ on an open interval extending left from $x_0$ and $f'(x) > 0$ on an open interval extending right from $x_0$ then $f$ has a relative minimum at $x_0$.

(c.) If $f'(x)$ has the same sign on an open interval extending left from $x_0$ as it does on an open interval extending right from $x_0$, then $f$ does not have a relative extremum at $x_0$.

## Second derivative test:

Suppose that $f$ is twice differentiable at the point $x_0$.

(a.) If $f'(x_0) = 0$ and $f''(x_0) > 0$ then $f$ has a relative minimum at $x_0$.

(b.) If $f'(x_0) = 0$ and $f''(x_0) < 0$ then $f$ has a relative ~~minimum at $x_0$~~ maximum at $x_0$.

(c.) If $f'(x_0) = 0$ and $f''(x_0) = 0$, then the test is inconclusive that is $f$ may have a relative maximum, a relative minimum or neither at $x_0$.

**Example:** Sketch the graph of the equation $y = x^3 - 3x + 2$ and identify the locations of the intercepts, relative extrema and inflection points.

**Solution:** $\because y = x^3 - 3x + 2 = (x+2)(x-1)^2$

**x-intercepts:** are $x = -2$ and $x = 1$

**y-intercept:** setting $x = 0 \Rightarrow y = 2$

**End behaviour:** $\lim\limits_{x \to +\infty} (x^3 - 3x + 2) = +\infty$

and $\lim\limits_{x \to -\infty} (x^3 - 3x + 2) = -\infty$

**Derivatives:** $\dfrac{dy}{dx} = 3x^2 - 3 = 3(x-1)(x+1)$

$\dfrac{d^2y}{dx^2} = 6x$

**Stationary Points:** $f'(x) = \dfrac{dy}{dx} = 0 \Rightarrow x = -1$ and $x = 1$.

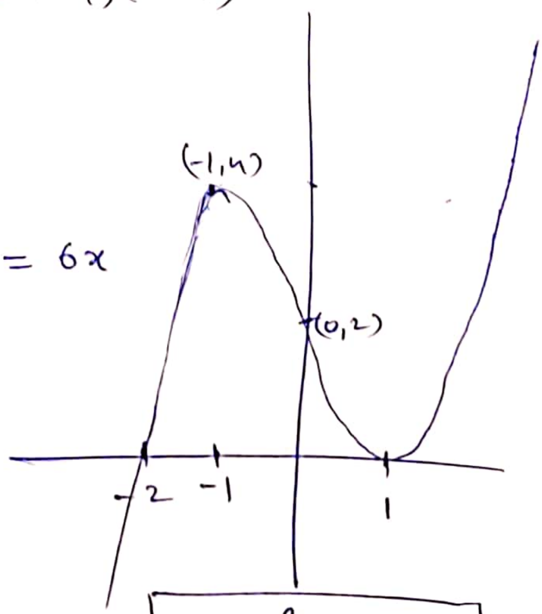
**Increase, Decrease, Relative extrema:**

| Increasing | | Decreasing | | Increasing |
|---|---|---|---|---|
| +ve | $-1$ | $-$ve | $1$ | +ve |

$\dfrac{dy}{dx} = 3(x-1)(x+1)$

| concave down | | concave up |
|---|---|---|
| $-$ve | $0$ | +ve |

$\dfrac{d^2y}{dx^2} = 6x$




$y = x^3 - 3x + 2$