Techniques of sketching conics : Parabola, Ellipse and hyperbola :

Question 7(a.) P-762 Sketch the parabola and level the focus, vertex and directrix

$$x^2 - 4x + 2y = 1 \quad \text{—— (1.)}$$

Solution.

$$x^2 - 4x + 4 = -2y + 1 + 4$$

$$\Rightarrow (x-2)^2 = -2\left(y - \tfrac{5}{2}\right) \quad \text{—— (2.)}$$

Compare this eqn. with $x^2 = -4pY$ —— (3.)

Axis of Symmetry: Since the eqn. of parabola involves $(x-2)^2$ term. Therefore the axis of symmetry is ~~along~~ parallel to y-axis. And the coefficient of $(y - \frac{5}{2})$ is negative, so that parabola opens downward. Axis of symm. is $x = 2$ line.

From (2.) & (3.), $4p = 2 \Rightarrow \boxed{p = \frac{1}{2}}$.

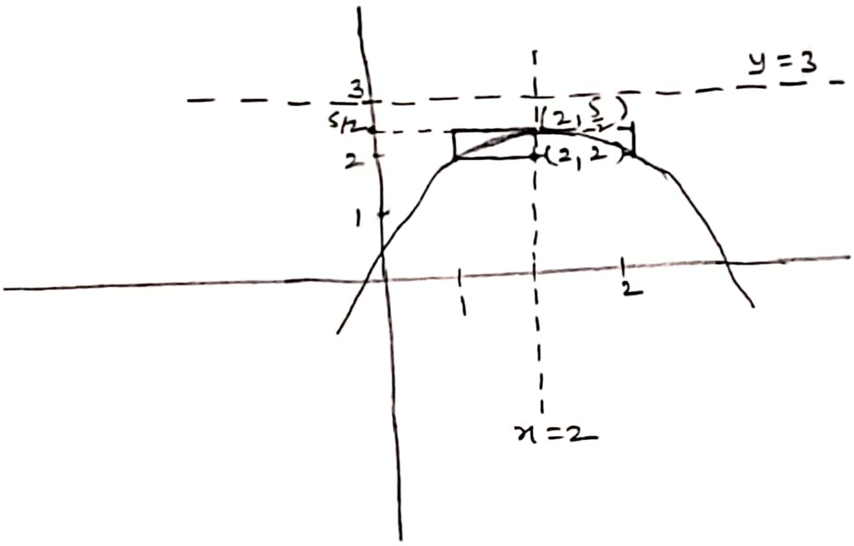
Vertex of the parabola is $(2, \frac{5}{2})$.

Now draw a box extending $p = \frac{1}{2}$ unit down from the vertex $(2, \frac{5}{2})$ and $2p = 1$ unit on each side of the axis of symmetry, $x = 2$.

Focus : Since focus is $p = \frac{1}{2}$ unit from the vertex $(2, \frac{5}{2})$ along the axis of symm. In downward direction. So its coordinates are $(2, 2)$.

Directrix : which is perpendicular to axis of symm. $x = 2$ and at a distance $p = \frac{1}{2}$ unit from the vertex on the opposite side from the focus i.e. $\boxed{y = \frac{5}{2} + \frac{1}{2}} = 3$ $\boxed{y = 3}$

Using the box as a guide, sketch the parabola so that its vertex is at $(2, \frac{5}{2})$ and it passes through the corners of the box.



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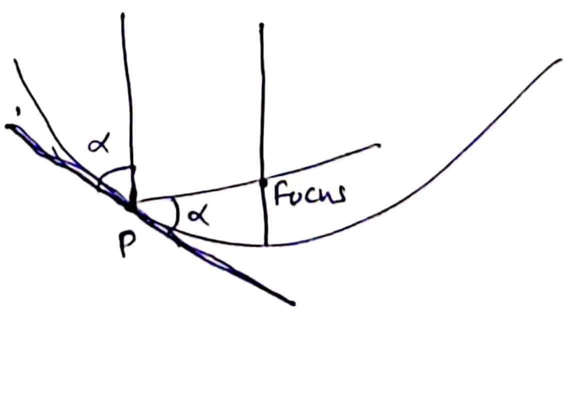
Reflection Property of Parabola:

The tangent line at a point P on a parabola makes equal angles with the line through P parallel to the axis of symmetry and the line through P and the focus.



Question. 26

P-762

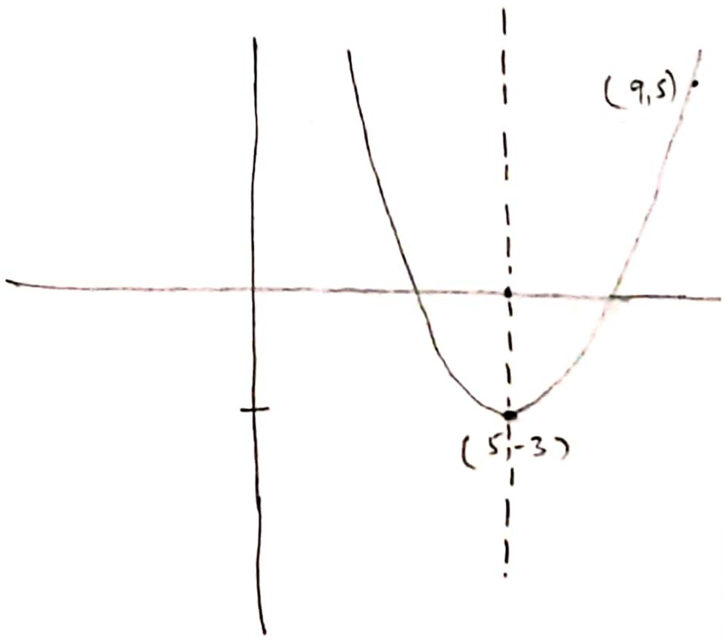
Find an eqn. for the parabola that has vertex $(5,-3)$; axis parallel to the y-axis; passes through $(9,5)$.

Solution. Since the parabola is symmetric about a line parallel to the y-axis and has its vertex at $(5,-3)$, the eqn. of the parabola is of the form

$$(x-5)^2 = 4p(y+3) \quad \text{or} \quad (x-5)^2 = -4p(y+3)$$

where the sign depends on whether the parabola opens up or down. But the parabola must open up, as it passes through the point $(9,5)$ which lies in the first quadrant while the vertex $(5,-3)$ lies in the fourth quadrant.

Thus the eqn. is of the form



$$(x-5)^2 = 4p(y+3)$$

$$(9-5)^2 = 4p(5+3)$$

$$\Rightarrow p = 2$$

$$\boxed{\therefore (x-5)^2 = 8(y+3)}$$

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