Subgroup:

A Subset H of G is called subgroup of G if H is itself a group under the operation of G.

Example: ① $G = (R, +)$ is a group

$H = (Q, +)$ is a subgroup of G.

②

$G = GL(2, R)$ under matrix multiplication is a group.

$H = SL(2, R)$ ~~under~~ is a subgroup of G.

$G_1 = \{1, -1, i, -i\}$ $i = \sqrt{-1}$.

G is group under multiplication.

$(1)^{-1} = 1$, $(-1)^{-1} = -1$, $(i)^{-1} = -i$, $(-i)^{-1} = i$

$H = \{1, -1\}$

Closed, Associative, Identity, Inverse.

H is a subgroup of G.

Order of a group

The no. of elements of a group is called its order. Notation: $|G|$ or $O(G)$.

$V(10) = \{1, 3, 7, 9\}$

$|V(10)| = 4$.

Order of an element:

The order of an element $g \in G$ is the smallest positive integer n such that

$$g^n = e. \quad \left[g^n = \underbrace{g * g * g * \cdots * g}_{n \text{ times}} \right]$$

In addition, $g^n = g + g + \cdots + g$

$= ng.$

If no such integer exist, we say that

Order of 2 is infinite.

Notation: $O(g)$ or $|g|$.

$U(12) = \{1, 5, 7, 11\}$, under multiplication modulo 12.

$|1| = 1$, $|5| = 2$. , 5

$5^1 = 5 \pmod{12} = 5$

$5^2 = 25 \pmod{12} = 1 = e$. $\because 5^4 = 1$

$\therefore |5| = 2$.

$7^1 = 7 \pmod{12} = 7$

$7^2 = 49 \pmod{12} = 1 \Rightarrow |7| = 2$

similarly $|11| = 2$.

(2) $G = \{1, -1, i, -i\}$, multiplication

Identity $= 1$.

$|1| = 1$, $|-1| = 2$, $|i| = 4$, $|-i| = 4$

(3) $(\mathbb{Z}_4, +_4)$ is a group

$\mathbb{Z}_4 = \{0, 1, 2, 3\}$, Identity $= 0$.

$|0| = 1$, $|+_4 1 +_4 1 +_4 1| = 4 \pmod 4 = 0$

$|1| = 4$, $2 +_4 2 = 4 \pmod 4 = 0$.

$|2| = 2$.

$|3| = 4$ $3 +_4 3 = 2$. $3 +_4 3 +_4 3 = 1$

$3 + a\ 3 + a\ 3 + a\ 3 = (2 \cos \angle a)$

$- = 0.$