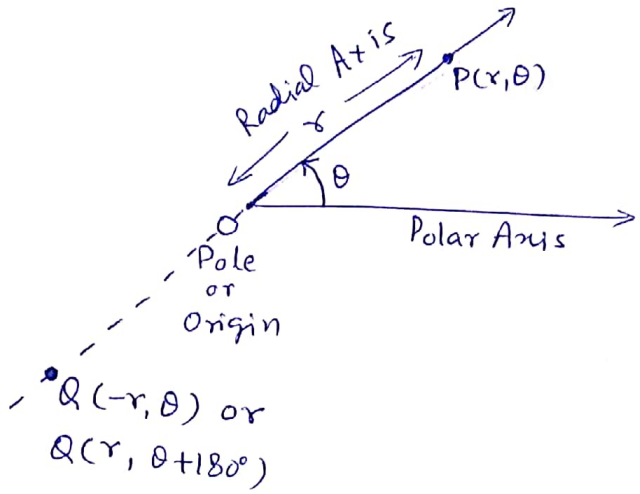
Polar coordinates:



The number r is called the radial coordinate of P and the number θ is called the angular coordinate or polar angle of P.

$(-r, \theta)$ and $(r, \theta+180^\circ)$ are polar coordinate of the same point.

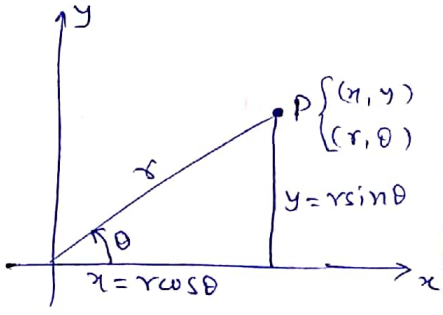
Relationship between Rectangular and Polar Coordinate:

(1.) To change from polar to rectangular:

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

(2.) To change from rectangular to polar:

$$r = \sqrt{x^2+y^2} \quad , \quad \tan\theta = \frac{y}{x} \quad \text{if}$$



Tracing a polar curve:

The following steps are helpful to trace a polar curve $r=f(\theta)$:

(1) **Symmetry Test:**

(i) A curve in polar coordinates is symmetric about the x-axis if replacing θ by $-\theta$ in its eqⁿ. produces an equivalent eqⁿ.

(ii) A curve in polar coordinates is symmetric about y-axis if replacing θ by $\pi-\theta$ in its eqⁿ. produces an equivalent eqⁿ.

(iii) A curve in polar coordinates is symmetric about the origin if replacing θ by $\pi+\theta$ or by replacing r by $-r$ in its eqⁿ. produces an equivalent eqⁿ.

(2) **Pole:** The curve passes through the pole if $r=0$ for a value of θ.

(3) **Region:** Find out the region on the plane in which no part of the curve lies.

(4) **Points:** Find different value of r for different values of θ.

Q.29 $r = 3(1-\sin\theta)$

Sketch the above graph in polar coordinates.

Soln: By (1.) Symmetry: By Replacing θ by $\pi-\theta$

$$r = 3[1-\sin(\pi-\theta)] = 3(1-\sin\theta)$$

$\therefore$ curve is symm. w.r.t. y-axis.

(2.) Pole: $r = 0 \Rightarrow 3(1-\sin\theta) = 0 \Rightarrow \sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$.

Thus the curve passes through the pole.

(3.) Region: As $-1 \le \sin\theta \le 1$

$$\therefore \quad 1-1 \le 1-\sin\theta \le 1+1$$

$$\Rightarrow \quad 0 \le 1-\sin\theta \le 2$$

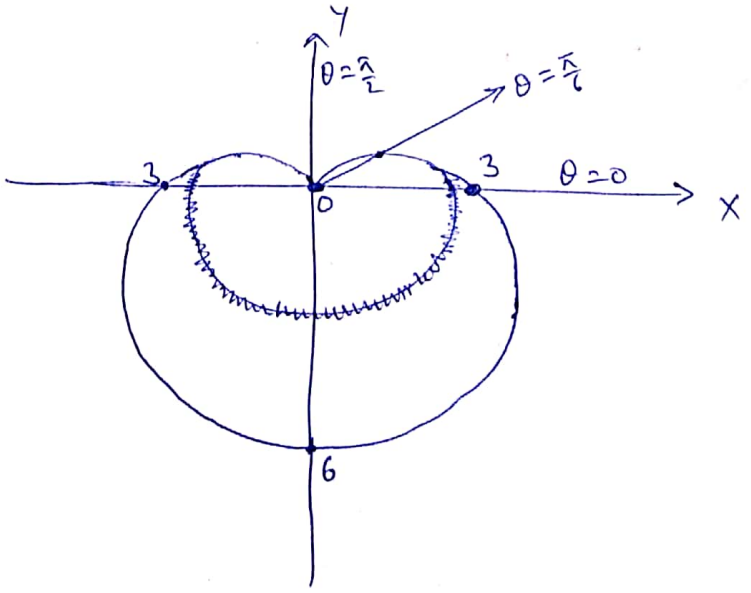
$$\Rightarrow \quad 0 \le 3(1-\sin\theta) \le 6$$

$$\text{or} \quad 0 \le r \le 6$$

$\therefore$ The curve lies within a circle of radius 6.

(4.) Points:

θ :	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r :	3	$\frac{3}{2}$	0	03	6	0



Tracing a polar curve:

The following steps are helpful to trace a polar curve $r = f(\theta)$

Q.35. Sketch the graph in polar coordinates:

$$r = 3 + \sin\theta$$

Soln (1.) **Symmetry:** Replacing θ by $\pi - \theta$

$$r = 3 + \sin(\pi - \theta) = 3 + \sin\theta$$

$\therefore$ The curve is symmetric w.r.t. y-axis

(2.) **Pole:** Put $r = 0 \Rightarrow 3 + \sin\theta = 0 \Rightarrow \sin\theta = -3$

which is not possible for any value of θ:

$\therefore$ Curve does not passes through pole.

(3.) **Region:** As $-1 \le \sin\theta \le 1 \Rightarrow 3 - 1 \le 3 + \sin\theta \le 3 + 1$

$$\Rightarrow 2 \le r \le 4$$

$\therefore$ The curve lies b/w the annulus of two circles of radius 2 and 4.

(4.) **Points:**

θ :	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
r :	3	$3+\frac{1}{2}$	4	3	2	3