Question 18. & Que^{n}. 17. A sphere has center in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3-\sqrt{3}$ units. What is the equation of the sphere?

Sol^{n}: Let $P(x_0, y_0, z_0)$ and r denote the center and radius of the sphere S, respectively.

In order for S to be tangent to the xy-plane, the distance $|z_0|$ from $P(x_0, y_0, z_0)$ to the xy-plane must equal r. Since $P(x_0, y_0, z_0)$ is in the first octant, we conclude that $z_0 = |z_0| = r$.

Similarly, $x_0 = y_0 = r$ and the center of S is $P(r, r, r)$.

Therefore the distance from the origin to the center $P(r,r,r)$ of S is $= \sqrt{r^2+r^2+r^2} = \sqrt{3}\, r$

Given that the distance from origin to the sphere S is $= 3 - \sqrt{3} = \sqrt{3}(\sqrt{3}-1)$ — (1.)

While the distance from origine to sphere S is $= \sqrt{3}\, r - r$ — (2.)

Equating (1.) & (2.) gives $(\sqrt{3}-1)r = \sqrt{3}(\sqrt{3}-1) \Rightarrow \boxed{r = \sqrt{3}}$

Therefore the coordinates of the center $P(r,r,r) = P(\sqrt{3},\sqrt{3},\sqrt{3})$ and eq^{n}. of sphere is $(x-\sqrt{3})^2+(y-\sqrt{3})^2+(z-\sqrt{3})^2 = 3$. □