Gradient :

vector differential operator 'del', written as $\vec{\nabla}$,

is defined as

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Let ϕ be a scalar funⁿ. having first order partial derivatives in some region. Then the gradient of ϕ, written as $\vec{\nabla}\phi$ or grad ϕ, is defined by

$$\text{grad}\,\phi = \boxed{\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}}.$$

$\vec{\nabla}\phi$ is a vector.

Geometrically, $\vec{\nabla}\phi$ is a vector normal to the surface $\phi(x,y,z) = c$, where c is a constant.

Divergence :

If $\vec{F}(x,y,z) = f(x,y,z)\,\hat{i} + g(x,y,z)\,\hat{j} + h(x,y,z)\,\hat{k}$

where f, g, h have first order partial derivatives

in some region, then we define the divergence

of $\vec{F}$, written as $\operatorname{div}\vec{F}$, by

$$\operatorname{div}\vec{F} = \vec{\nabla}\cdot\vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(f\hat{i} + g\hat{j} + h\hat{k}\right)$$

$$\boxed{\vec{\nabla}\cdot\vec{F} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}}$$

Note that $\vec{\nabla}\cdot\vec{F}$ is a scalar fu^n and $\vec{\nabla}\cdot\vec{F} \neq \vec{F}\cdot\vec{\nabla}$.

If $\vec{a}$ is a constant vector, then $\operatorname{div}\vec{a} = 0$.

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ where a_1, a_2, a_3 are constants.

$$\therefore \quad \operatorname{div}\vec{a} = \frac{\partial}{\partial x}(a_1) + \frac{\partial}{\partial y}(a_2) + \frac{\partial}{\partial z}(a_3) = 0+0+0 = 0$$

Solenoidal Vector :

A vector $\vec{A}$ is called solenoidal if its divergence is zero i.e.

$$\vec{\nabla}\cdot\vec{A} = 0$$

Curl:

If $\vec{F}(x,y,z) = f(x,y,z)\hat{i} + g(x,y,z)\hat{j} + h(x,y,z)\hat{k}$,

where f, g, h have first order partial derivatives in some region, then we define the curl of $\vec{F}$, written as curl $\vec{F}$, by

$$\text{curl } \vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & g & h \end{vmatrix}$$

$$\text{curl } \vec{F} = \left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right)\hat{i} + \left(\frac{\partial f}{\partial z} - \frac{\partial h}{\partial x}\right)\hat{j} + \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)\hat{k}$$

Note that curl $\vec{F}$ is a vector.

If $\vec{a}$ is a constant vector, then curl $\vec{a} = \vec{\nabla} \times \vec{a} = 0$.

Irrotational Vector:

A vector $\vec{A}$ is called irrotational if its curl is zero i.e.

$$\vec{\nabla} \times \vec{A} = 0$$

Laplacian Operator:

$$\vec{\nabla}^2 = \vec{\nabla} \cdot \vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\vec{\nabla}^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$, where $\phi(x,y,z)$ is some